CONTRASTIVE EVIDENCE IN THE AGM FRAMEWORK

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1. Preliminaries

$\mathcal{L}$: arbitrary propositional ('factual') language, constructed by means of the standard Boolean connectives $\{\vee, \wedge, \neg, \rightarrow\}$

$\mathcal{L}_E$: smallest extension of $\mathcal{L}$ that includes all sentences of the form $\varphi \rhd \psi$ and $\varphi \rhd_\chi \psi$ (where $\varphi, \psi, \chi \in \mathcal{L}$)

K: arbitrary belief set, a subset of $\mathcal{L}_E$

$\mathbb{K}$: set of all rationally permissible belief sets

M: arbitrary belief revision model, a pair of functions $\langle *, \dot{-} \rangle$, such that $*$ (the revision function) and $\dot{-}$ (the contraction function) map elements of $\mathbb{K} \times \mathcal{L}$ onto $\mathbb{K}$

$\mathbb{M}$: set of all rationally permissible belief revision models

Cn: consequence operator, mapping subsets of $\mathcal{L}_E$ onto other such subsets

2. Principles

Consequence:

> **(DT)** For all $A, B \in \mathcal{L}$ and $\Gamma \subseteq \mathcal{L}_E$, if $B \in \text{Cn}(\Gamma \cup \{A\})$, then $A \rightarrow B \in \text{Cn}(\Gamma)$

(SUP) For all $\Gamma \subseteq \mathcal{L}_E$, $\text{Cn}_0(\Gamma_0) \subseteq \text{Cn}(\Gamma)$, where Γ_0 denotes the factual subset of Γ and Cn_0 returns the set of its classical consequences

Belief sets:

(CL) For all $K \in \mathbb{K}$, $\text{Cn}(K) \subseteq K$

(CON) For all $K \in \mathbb{K}$ and $A \in \mathcal{L}$, there is no finite $\Gamma \subseteq K$ such that $A \wedge \neg A \in \text{Cn}(\Gamma)$.

Revision & contraction:[1]

(AGM*2) If $\neg A \notin \text{Cn}(\varnothing)$, then $A \in K * A$.

(AGM*3) If $B \in K * A$, then $B \in \text{Cn}(K \cup \{A\})$

(AGM*V) If $\neg A \in \text{Cn}(\varnothing)$ then $K * A = K$.

(AGM*6) If $\text{Cn}(A) = \text{Cn}(B)$, then $K * A = K * B$

(AGM*7) $K * (A \wedge B) \subseteq \text{Cn}(K * A \cup \{B\})$

(CM) If $B \in K * A$ and $C \in K * A$, then $C \in K * A \wedge B$

(I*1) If $C \in \text{Cn}(A)$, then $B \in K * A$ iff $B \in (K * C) * A$

(I*3) If $B \in K * A$, then $B \in (K * B) * A$

(AGM $\dot{-}$2) If $B \in K \dot{-} A$, then $B \in K$

(AGM $\dot{-}$3) If $A \notin K$ or $A \in \text{Cn}(\varnothing)$, then $K \dot{-} A = K$

(AGM $\dot{-}$4) If $A \notin \text{Cn}(\varnothing)$, then $A \notin K \dot{-} A$

(AGM $\dot{-}$5) If $\text{Cn}(A) = \text{Cn}(B)$, then $K \dot{-} A = K \dot{-} B$

(AGM $\dot{-}$6) If $B \in K$, then $B \in \text{Cn}((K \dot{-} A) \cup \{A\})$

(I $\dot{-}$2) If $C \in \text{Cn}(A)$, then $B \in K * A$ iff $B \in (K \dot{-} C) * A$

(I $\dot{-}$4) If $B \notin K * A$, then $B \notin (K \dot{-} B) * A$

(I $\dot{-}$4+) If $B \in \text{Cn}(C)$, then if $B \notin (K \dot{-} C) * A$, then $B \notin (K \dot{-} B) * A$[2]

[1]All principles are to be read as holding for all $K \in \mathbb{K}$, $A, B, C \in \mathcal{L}$ and $*$, $\dot{-}$ that are members of some $M \in \mathbb{M}$.

[2]Entails (I $\dot{-}$4).

Evidential beliefs:

(BEV) $A \rhd B \in K$ iff $B \in (K \dot{-} B) * A$

(TEV) Where $\neg C \in \text{Cn}(B)$, $A \rhd_C B \in K$ iff $B \in ((K \dot{-} \neg C) * B \vee C) * A$

(TR1) If $C \in \text{Cn}(B)$, then $A \rhd C \in \text{Cn}(A \rhd B)$.

(TR1') If $C \in \text{Cn}(B)$ and $A \rhd B \in K$, then $A \rhd C \in K$.

3. Observations and selected proofs

Obs1: Given (CL), (CON), (DT), (AGM*2), (AGM*3), (AGM*6), (AGM*7), (AGM $\dot{-}$3), (AGM $\dot{-}$5) and (I $\dot{-}$4+), (BEV) entails:

(ABS) If $A \rhd B \in K$, then $A \rhd (A \wedge B) \in K$

(CA) $A \rhd C \in K$ and $B \rhd C \in K$, then $(A \vee B) \rhd C \in K$

(CC) If $A \rhd B \in K$ and $A \rhd C \in K$, then $A \rhd (B \wedge C) \in K$

(CN) If $\neg A \notin \text{Cn}(\varnothing)$ and $B \in \text{Cn}(\varnothing)$, then $A \rhd B \in K$

(ID) If $A \notin \text{Cn}(\varnothing)$, then $A \rhd A \in K$

(LLE) If $(A \leftrightarrow C) \in \text{Cn}(\varnothing)$, then $A \rhd B \in K$ iff $C \rhd B \in K$.

(MP) If $A \rhd B \in K$ then $A \rightarrow B \in K$

(NEC) If $\neg B \in \text{Cn}(\varnothing)$ then $A \rhd B \notin K$

(RLE) If $(B \leftrightarrow C) \in \text{Cn}(\varnothing)$, then $A \rhd B \in K$ iff $A \rhd C \in K$

(RRT) If $B \notin K$, then $A \rhd B \in K$ iff $B \in K * A$

(SCC) If $A \rhd B \in K$ and $\neg B \vee \neg C \in \text{Cn}(\varnothing)$, then $A \rhd C \notin K$

(SUP$\rhd$) If $A \notin \text{Cn}(\varnothing)$ and $B \in \text{Cn}(A)$, then $A \rhd B \in K$

Proof: Available on request.

Obs2: Given (CL), (CON), (AGM*2), (AGM*3), (AGM $\dot{-}$ 4) and (I $\dot{-}$4+), (BEV) is inconsistent, on pains of various triviality results, with:

(CCC) If $A \rhd B \in K$ and $B \in \text{Cn}(C)$, then $A \rhd C \in K$

(CS) If $A, B \in K$, then $A \triangleright B \in K$

(EEM) Either $A \triangleright B \in K$ or $A \triangleright \neg B \in K$

(SYM) If $A \triangleright B \in K$, then $B \triangleright A \in K$

Proof: Available on request.

Obs3: (BEV), in the presence of (TR1'), (CL), (SUP), (AGM*2), and (AGM*4) entails:

(TRIV) There is no $K \in \mathbb{K}$, and $A, B, C \in \mathcal{L}$ such that $A \in K$, $B, \neg B \notin K$ but $B \triangleright A \notin K$.

Proof: See Chandler (2012), fn. 27.

Obs4: Given (AGM * 2), (AGM * 3), (AGM * V), (CM), (I*1), (AGM $\dot{-}$ 2), (AGM $\dot{-}$ 3), (AGM $\dot{-}$ 4), (AGM $\dot{-}$ 6), (I $\dot{-}$2), (I $\dot{-}$4), (CL), (DT) and (SUP), (BEV) entails

(INT1) If $C \in \text{Cn}(B)$ and $A \triangleright B \in K$, then $A \triangleright C \notin K$ iff $A \triangleright B \notin K \dot{-} C$.

Proof: See Chandler (2012), proof of Observation 5.

Obs5: Given (AGM*V), (BEV) and (TEV) jointly entail that $A \triangleright B \in K$ iff $A \triangleright_{\neg B} B \in K$.

Proof: By (SUP), $\text{Cn}(B) = \text{Cn}(\neg\neg B)$, so by (AGM $\dot{-}$6) $(K \dot{-} B) = (K \dot{-} \neg\neg B)$. Furthermore, by (SUP) again, $B \vee \neg B \in \text{Cn}(\varnothing)$, so by (AGM*V), $(K \dot{-} \neg\neg B) = (K \dot{-} \neg\neg B) * B \vee \neg B$. Putting this together, $(K \dot{-} B) = (K \dot{-} \neg\neg B) * B \vee \neg B$, and hence $(K \dot{-} B) * A = ((K \dot{-} \neg\neg B) * B \vee \neg B) * A$. By (BEV) and (TEV), this is equivalent to $A \triangleright B \in K$ iff $A \triangleright_{\neg B} B \in K$. QED.

Obs6: Given (DT), (CL), (I*1), (I*3), (AGM*7), (CM), (I $\dot{-}$2) and (I $\dot{-}$4), (TEV) entails:

(INT2) If $C \in \text{Cn}(B)$, $A \triangleright B \in K$, then (1) $A \triangleright B \notin K \dot{-} C$ iff (2) $A \triangleright_{\neg C} B \notin K$.

Proof: We first note that the following principle, which we shall occasionally use later on, is a well-known weakening of (AGM*7):

(CUT) If $B \in K * A$, then $K * (A \wedge B) \subseteq K * A$

With this in hand, we now assume $C \in \mathrm{Cn}(B)$. We first establish the following lemma:

Lemma 1: If (i) $B \in K * A$, then (ii) $K * A = (K * C) * A$

Assume (i) for conditional proof. By (i), the fact that $C \in \mathrm{Cn}(B)$ and (CL), it follows that

$$C \in K * A \tag{1}$$

From (1), it then follows by (CM) and (CUT) that

$$K * A = K * A \wedge C \tag{2}$$

From (1), but this time by (DP*3), we obtain

$$C \in (K * C) * A \tag{3}$$

And, from (3), it then follows by (CM) and (CUT) that

$$(K * C) * A \wedge C = (K * C) * A \tag{4}$$

Finally, since $C \in \mathrm{Cn}(A \wedge C)$, it follows from (DP*1) that

$$K * A \wedge C = (K * C) * A \wedge C \tag{5}$$

Putting together (2), (4) and (5), we recover the desired result that (ii) $K * A = (K * C) * A$. This completes the proof of Lemma 1.

We now derive a second lemma:

Lemma 2: If (i) $A \triangleright B \in K$, then (ii) $B \in (K \dot{-} C) * A \wedge C$[3]

Assume (i) for conditional proof. It follows, by the left-to-right direction of (BEV), that

$$B \in (K \dot{-} B) * A \tag{6}$$

From (6) , and the fact that $C \in \mathrm{Cn}(B)$, it follows by (CL) that

$$C \in (K \dot{-} B) * A \tag{7}$$

From (6), (7) and (CM), it then follows that

$$B \in (K \dot{-} B) * A \wedge C \tag{8}$$

[3]This is Lemma 1 of the proof of Observation 5 in Chandler (2012).

Now assume for reductio that $B \notin (K \dot{-} C) * A \wedge C$. Since $C \in \mathrm{Cn}(A \wedge C)$, it follows by (I $\dot{-}$2) that

$$B \notin K * A \wedge C \tag{9}$$

By (9) and (I $\dot{-}$4), it then follows that

$$B \notin (K \dot{-} B) * A \wedge C \tag{10}$$

Contradiction. Hence (ii): $B \in (K \dot{-} C) * A \wedge C$. This completes the proof of Lemma 2.

Finally, we derive a third and final lemma:

> *Lemma 3:* If $B \in (K \dot{-} C) * A \wedge C$, then $A \rhd B \notin K \dot{-} C$ iff $A \rhd_{\neg C} B \notin K$

Assume $B \in (K \dot{-} C) * A \wedge C$ for conditional proof. By (AGM*7)

$$B \in \mathrm{Cn}((K \dot{-} C) * A \cup \{C\}) \tag{11}$$

It follows from (11), by the deduction theorem (DT), which Cn satisfies, that

$$B \vee \neg C \in (K \dot{-} C) * A \tag{12}$$

Obviously, $B \vee \neg C \in \mathrm{Cn}(B \vee \neg C)$. So by (12) and Lemma 1, we have $(K \dot{-} C) * A = ((K \dot{-} C) * B \vee \neg C) * A$ and hence

$$B \in (K \dot{-} C) * A \text{ iff} B \in ((K \dot{-} C) * B \vee \neg C) * A \tag{13}$$

By (BEV) and (TEV), (13) is equivalent to:

$$A \rhd B \notin K \dot{-} C \text{ iff} A \rhd_{\neg C} B \notin K \tag{14}$$

This completes the proof of Lemma 3.

Obs6 then follows from the conjunction of Lemmas 2 and 3. QED.

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