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[10] Joyce's Accuracy-Based Argument (ctd.) + Indifference

0. Outline

1. **Joyce's 'accuracy-based' argument**
2. **The Principle of Indifference**

1. Joyce's 'accuracy-based' argument

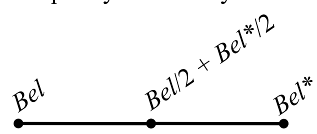
- Last session:
  - Joyce wants an 'epistemic' vindication of probabilism (PROB).
  - He starts off with: rational agents have belief functions that minimise inaccuracy, i.e. have maximally strong degrees of belief in truths and minimally strong degrees of belief in falsehoods
  - He then attempts to provide constraints on any adequate inaccuracy function *I*.
  - We saw two such constraints: **Dominance** and **Normality**.
  - These two are pretty uncontroversial.

1. Joyce's 'accuracy-based' argument

- Third principle:

  Weak Convexity: If $I(Bel, w) = I(Bel^*, w)$, then $I(Bel, w) \geq I(Bel/2 + Bel^*/2, w)$, with $I(Bel, w) = I(Bel/2 + Bel^*/2, w)$ iff $Bel = Bel^*$.

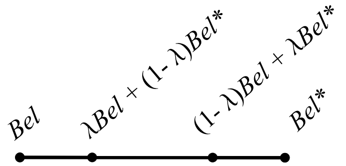
  What this means: if two belief functions are equally inaccurate, then the belief function formed by averaging the two functions is at least as accurate as either of those functions, with equality if and only if the functions are identical.



1. Joyce's 'accuracy-based' argument

- Fourth and final principle:

  Symmetry: If $I(Bel, w) = I(Bel^*, w)$, then $I(\lambda Bel + (1\text{-}\lambda)Bel^*, w) = I((1\text{-}\lambda)Bel + \lambda Bel^*, w)$, for any $0 \leq \lambda \leq 1$.



1. Joyce's 'accuracy-based' argument

- With this in hand, Joyce proves that, assuming that I satisfies D:

  (a) if *Bel* isn't a probability function then there exists a belief function *Bel** that *is* a probability function and is such that for every $w \in \Omega$, $I(Bel, w) > I(Bel^*, w)$.
- He also tells us that he has a proof for the following. Apparently (according to Hajek [forth.]), this involves placing an additional requirement on I that Joyce calls '**Propriety**':

  (b) if *Bel* *is* a probability function then there *doesn't* exist a belief function *Bel** such that for every $w \in \Omega$, $I(Bel, w) > I(Bel^*, w)$.
- Joyce then concludes that PROB is true, providing, according to him, a purely epistemic vindication of probabilism.

1. Joyce's 'accuracy-based' argument

- The recent literature discussing Joyce's argument is a little technical for this course.
- Maher [2002], however, is quite accessible.
- Some issues he raises:
  - Joyce [1998] failed to motivate either **Weak Convexity** (invalid argument) or **Symmetry** (valid but unsound argument), and without them, the desired results don't follow.
  - The following measure is, according to him, a perfectly intuitive measure of inaccuracy but conflicts with both **Weak Convexity** and **Symmetry** (although is compatible with the previous two requirements):

$$\sum_i \left| Bel(P_i) - T(P_i, w) \right|$$

2. The Principle of Indifference

- We have seen one synchronic constraint on d.o.b.s: probabilistic coherence.
- Another popular synchronic constrain: the Principle of Indifference.
- This time it connects the d.o,b.s of an agent with her epistemic position.
- It goes something like this (there are many different versions):

  PI: if *S* isn't epistemically justified in preferring to believe any proposition over any other in some finite partition of Ω, then *S* should assign equal degrees of belief to each member of that partition.
- Proponents: Laplace, Bernouilli, Leibniz, Pascal, Keynes, etc.

2. The Principle of Indifference

- Where n denotes the number of cells in the partition, according to PI, S is therefore rationally compelled to have $\text{Bel}_S(P_i) = 1/n$ for $1 \leq i \leq n$ (on pains of violating [P2] or [P3]).
- This seems on the face of it to yield some very rationally compelling verdicts in various simple cases.
- E.g.: if I don't know whether a coin will turn up heads or tails I might say that my balance of confidence is 'fifty/fifty'.
- From this we can then compute S's rational d.o.b. in the union of m members of the partition: m/n.
- Now what about the countably infinite case (i.e. a partition of Ω into countably infinitely many cells)?
- We can extend PI, but *only if* we reject countable additivity.

2. The Principle of Indifference

- If we do reject countable additivity, S is then rationally compelled to set $\text{Bel}_S(P_i) = 0$ for $i = 1, 2, 3, \ldots$ (see L3)
- Note: those who don't reject countable additivity sometimes offer a generalisation of PI that also makes recommendations for the countably infinite case (albeit different recommendations).
- It appeals to the notion of *entropy*, which, in this context, measures the 'flatness' of a belief function.
- It goes like this:

  MaxEnt: if S isn't epistemically justified in preferring to believe any proposition over any other in some partition of Ω, then S's belief function should maximise entropy. The entropy of Bel is given by: $-\sum_i Bel_S(P_i)\log_2 Bel_S(P_i)$

2. The Principle of Indifference

- **PI**, it turns out, falls out as a special case of **MaxEnt**.
- All this sounds quite sensible…
- Problem: **PI** is widely known to lead to a well-known stock of paradoxes.
- A not-so-tricky one:
  - An urn contains white and coloured balls in proportions unknown to S.
  - S isn't epistemically justified in preferring to believe any proposition over any other in the following finite partition P of Ω:

    $P = \{W, C\}$ (where W = next ball drawn will be white, and C = next ball drawn will be coloured)

2. The Principle of Indifference

- Therefore, by **PI**, S should have $\text{Bel}_S(W) = \text{Bel}_S(C) = ½$
- But now consider: the set of coloured balls is *itself* made up of red and blue balls in proportions unknown to S.
- S isn't epistemically justified in preferring to believe any proposition over any other in the following finite partition P^* of Ω: :

  $P^* = \{W, R, B\}$ (where W = next ball drawn will be white, R = next ball drawn will be red, and B = next ball drawn will be blue).
- Therefore, by **PI**, $\text{Bel}_S(W) = \text{Bel}_S(R) = \text{Bel}_S(B) = 1/3$.
- Now **PI** yields contradictory prescriptions: S should both have $\text{Bel}_S(W) = 1/2$ and $\text{Bel}_S(W) = 1/3$.

2. The Principle of Indifference

- This case may not be too difficult to solve.
- P and P^* are after all different partitions of Ω and we may wish to argue that there is a principled way of picking out a privileged partition wrt which **PI** applies.
- This seemed to be one of Keynes' suggestions (Gillies [2000:42]).
- But (many!) problem cases remain, in which contradictory applications of **PI** do *not* rest on different manners of carving up the outcome space.
- Some definitions:
  - Informally, a *random variable X* is a quantity that can take on different values in different possible worlds, i.e. a function from possible worlds to a set of numbers.

2. The Principle of Indifference

E.g.: the weight W of the average Wotsit produced in Willie Wonka's factory.

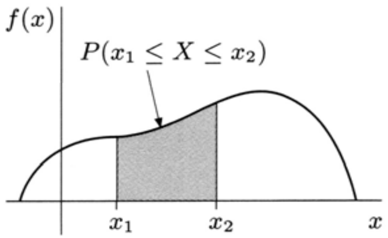
E.g.: our 'truth' function $T(P, \cdot)$.

- A *continuous* random variable is a random variable that can take on *uncountably* many values (e.g. any value in some non-degenerate interval of the reals).

E.g: W is a continuous r.v. $T(P, \cdot)$ is not (it is a *discrete* r.v.).

Where X is a continuous r.v., a *probability density function* is (roughly) a function f such that the probability that $x_1 \leq X \leq x_2$ is given by the area under the graph of f between x_1 and x_2. (i.e.: $\Pr(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f(x)dx$, the integral of f on $[x_1, x_2]$)

2. The Principle of Indifference



Reference

- Gillies, D. [2000]: *Philosophical Theories of Probability*. London: Routledge.
- Hajek , A. [forth.] 'Arguments for - or against - probabilism', forthcoming in Huber and Schmidt-Petri (eds.) *Degrees of Belief*.
- Maher, P. [2002]: 'Joyce's Argument for Probabilism', *Philosophy of Science* 69: 73-81.

Next lecture: 'Indifference (ctd.) + Updating Belief'

- No set reading.
- You might find the following helpful (*very* clear introductory overview of the paradoxes just discussed):
  - Gillies, D. [2000]: *Philosophical Theories of Probability*. London: Routledge. Pages 37-49.