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[7] Synchronic Dutch Book Arguments (ctd.)

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BELIEF & INQUIRY

0. Outline

1. The Synchronic Dutch Book Theorem
2. Completing the Argument
3. Objections (1): 'Czech' books

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1. The Synchronic Dutch Book Theorem

- Last time:
 - Betting terminology
 - BET
 - Proof that if BET and $\text{Bel}_S(P) < 0$, then DB.
 - Proof that if BET and $\text{Bel}_S(\Omega) \neq 1$, then DB.
 - Proof that if BET, $P \cap Q = \emptyset$ and $\text{Bel}_S(P \cup Q) \neq \text{Bel}_S(P) + \text{Bel}_S(Q)$, then DB.
- Well how about *countable* additivity (i.e. [P3])?
- Well it turns out that if countable additivity is violated, then DB.
- The proof is (unsurprisingly) similar to that given for finite additivity.

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1. The Synchronic Dutch Book Theorem

- I'll give you a quick sketch from Bartha [2004: 306], for the special case in which we consider a denumerable partition $\{P_1, P_2, \dots\}$ of Ω , rather than a partition of some other event.
- We want to show that if $\text{Bel}_S(P_1) + \text{Bel}_S(P_2) + \dots \neq \text{Bel}_S(\Omega)$ then DB.
- We have two cases to consider: (i) $>$ and (ii) $<$.
- Now it happens to follow from finite additivity ([P3]) that $\text{Bel}_S(P_1) + \text{Bel}_S(P_2) + \dots \leq \text{Bel}_S(\Omega)$.
- So we just need to consider $\text{Bel}_S(P_1) + \text{Bel}_S(P_2) + \dots < \text{Bel}_S(\Omega) = 1$ (that $\text{Bel}_S(\Omega) = 1$ follows from [P2])
- Assume that this is the case.
- Now offer S a denumerable set of individual bets wrt each P_i .

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1. The Synchronic Dutch Book Theorem

- Set equal stakes of - 1 for each bet.
- For each bet, we get the following payoff table:

P_i	$\ \langle P_i, \text{Bel}_S(P_i), -1 \rangle\ $
T	$\text{Bel}_S(P_i) - 1$
F	$\text{Bel}_S(P_i)$

- Now call the cell of the partition that is true (exactly one of them has to be: we are considering a partition of Ω) P_n .
- S gets a payoff of $\text{Bel}_S(P_i)$ for each bet against a P_i that isn't P_n , as these P_i 's are all false.
- S also gets $\text{Bel}_S(P_n) - 1$ for his bet against P_n .
- Total payoff: $\text{Bel}_S(P_1) + \text{Bel}_S(P_2) + \dots - 1 < 0$. Dutch book. ■

1. The Synchronic Dutch Book Theorem

- For the part of the theorem pertaining to [P4], we need to introduce an additional concept:

A *conditional bet* $\langle P|Q, \mathcal{S}, q \rangle$, is a bet on whether or not P that is called off – i.e. you get your expenditure back – iff Q is false.

Here is the payoff table:

P	Q	$\ \langle P Q, q, \mathcal{S} \rangle\ $
T	T	$\mathcal{S} - q\mathcal{S}$
T	F	0
F	T	$-q\mathcal{S}$
F	F	0

- Obviously, this is a generalisation of our concept of unconditional bet, which falls out as the special case in which the bet is conditional on Ω (i.e. $\langle P, \mathcal{S}, q \rangle = \langle P|\Omega, \mathcal{S}, q \rangle$).

1. The Synchronic Dutch Book Theorem

- **DUTCH₄**: assuming BET and $\text{Bel}_S(Q) = q_1 > 0$, if $\text{Bel}_S(P|Q) = q_3 \neq \text{Bel}_S(P \cap Q) / \text{Bel}_S(Q) = q_2 / q_1$ then Dutch book.

Proof: Here again, we have to consider a combination of bets: a bet of whether or not Q , a bet on whether or not P conditional on Q and a bet on whether or not $P \cap Q$.

Payoff table:

P	Q	$P \cap Q$	$\ \langle Q, q_1, \mathcal{S}_1 \rangle\ $	$\ \langle P Q, q_3, \mathcal{S}_3 \rangle\ $	$\ \langle P \cap Q, q_2, \mathcal{S}_2 \rangle\ $
T	T	T	$\mathcal{S}_1 - q_1\mathcal{S}_1$	$\mathcal{S}_3 - \mathcal{S}_3q_3$	$\mathcal{S}_2 - q_2\mathcal{S}_2$
T	F	F	$-q_1\mathcal{S}_1$	0	$-q_2\mathcal{S}_2$
F	T	F	$-\mathcal{S}_1 - q_1\mathcal{S}_1$	$-\mathcal{S}_3q_3$	$-q_2\mathcal{S}_2$
F	F	F	$-q_1\mathcal{S}_1$	0	$-q_2\mathcal{S}_2$

1. The Synchronic Dutch Book Theorem

We again divide the proof into two steps: (i) assuming $q_1 > 0$, if $q_3 > q_2 / q_1$ then Dutch book, (ii) assuming $q_1 > 0$, if $q_3 < q_2 / q_1$ then Dutch book.

- (i) (i.e. assuming $q_1 > 0$, if $q_3 > q_2 / q_1$ then DB). Let's set:

the stake for the bet on whether or not Q to $\text{Bel}_S(P|Q) = q_3$

the stake for the bet on whether or not P conditional on Q to 1

the stake for the bet on whether or not $P \cap Q$ to - 1

P	Q	$P \cap Q$	$\ \langle Q, q_1, q_3 \rangle\ $	$\ \langle P Q, q_3, 1 \rangle\ $	$\ \langle P \cap Q, q_2, -1 \rangle\ $
T	T	T	$q_3 - q_1q_3$	$1 - q_3$	$-1 + q_2$
T	F	F	$-q_1q_3$	0	q_2
F	T	F	$q_3 - q_1q_3$	$-q_3$	q_2
F	F	F	$-q_1q_3$	0	q_2

1. The Synchronic Dutch Book Theorem

Whatever the combination of truth values for P and for Q , the payoff for the set of bets is $q_2 - q_1q_3$. Now, on the assumption that $q_1 > 0$ and $q_3 > q_2 / q_1$ (and hence $q_2 < q_1q_3$), this quantity will be negative. Dutch book. ■

(ii) (i.e. assuming $q_1 > 0$, if $q_3 < q_2 / q_1$ then DB). Exact same procedure as above, except that this time we set:

the stake for the bet on whether or not Q to $-\text{Bel}_S(P|Q) = -q_3$

the stake for the bet on whether or not P conditional on Q to -1

the stake for the bet on whether or not $P \cap Q$ to 1 .

1. The Synchronic Dutch Book Theorem

P	Q	$P \cap Q$	$\ \langle Q, q_1, -q_3 \rangle \ $	$\ \langle P Q, q_3, -1 \rangle \ $	$\ \langle P \cap Q, q_2, 1 \rangle \ $
T	T	T	$q_1q_3 - q_3$	$q_3 - 1$	$q_2 - 1$
T	F	F	q_1q_3	0	$-q_2$
F	T	F	$q_1q_3 - q_3$	$q_3 - 1$	$-q_2$
F	F	F	q_1q_3	0	$-q_2$

This time, whatever the combination of truth values for P and for Q , the payoff for the set of bets is $q_1q_3 - q_2$. Now, on the assumption that $q_1 > 0$ and $q_3 < q_2 / q_1$ (and hence $q_2 > q_1q_3$), this quantity will again be negative. Dutch book. ■

▪ Voila!

2. Completing the Argument

▪ So far, we have:

(i) BET: If $\text{Bel}(P) = q$, then for any relatively small $|\delta|$, S would be happy to accept any bet $B = \langle P, q, \delta \rangle$, q being known as S 's fair betting quotient.

(ii) DUTCH: Assuming BET, if S 's belief function violates any of [P1], [P2], [P3], [P3*] or [P4], then there exists a set of bets that S would regard as fair, such that that set of bets would guarantee S a sure loss.

▪ Many take this vulnerability to sure losses to be sufficient for S to be irrational, adding the following premise:

(iii) If there exists a set of bets that S would regard as fair, such that accepting that set of bets would guarantee S a sure loss, then S is irrational.

2. Completing the Argument

▪ So we end up with:

∴ (iv) PROB: If S 's belief function violates any of [P1], [P2], [P3], [P3*] or [P4], then S is irrational.

▪ But step (iii) is problematic: entailing an unfortunate state of affairs surely isn't sufficient for a choice (here: prob. incoherence over coherence) to be irrational – you could be damned if you do and damned if you don't (see e.g. Hajek [forthcoming]).

▪ Consider:

(i) If you went to the SH reading party this week, you could die tomorrow.

(ii) Dying tomorrow is an unfortunate s.o.a.

∴ (iii) It was irrational for you to go to the SH reading party.

2. Completing the Argument

- Obviously a non sequitur: we would need to establish that possibly dying tomorrow *isn't* also entailed by *not* having been to the reading party.
- So to be even prima facie convincing, the DBA needs something further.
- This something further is known as the *converse* synchronic Dutch book theorem:

CDUTCH: Assuming BET, if *S*'s belief function *doesn't* violate any of [P1], [P2], [P3], [P3*] or [P4], then there *doesn't* exist a set of bets that *S* would regard as fair, such that accepting that set of bets would guarantee *S* a sure loss.
- I'll spare you the proof for this (see Kemeny [1955] if you are interested).

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2. Completing the Argument

- We can now rerun an altogether more convincing DBA, adding CDUTCH as a premise:
 - (i) BET
 - (ii) DUTCH
 - (iii) CDUTCH
 - (iv) If (a) there exists a set of bets that *S* would regard as fair, such that accepting that set of bets would guarantee *S* a sure loss if and only if *S*'s belief function isn't a (countably additive) probability function, then (b) unless *S*'s belief function is a probability function, *S* is irrational.

∴ (v) PROB: unless *S*'s belief function is a (countably additive) probability function, *S* is irrational.

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2. Completing the Argument

- Now note that, interestingly, whilst we saw one argument *against* requiring [P3*] in L3/L4, we now appear to have a second argument *running the other way*.
- Williamson [1999] takes the DBA for countable additivity to decisively require dropping De Finetti's claim that it is rationally permissible to have equal d.o.b.'s in the members of a countably infinite partition.
- Although the synchronic DBA may look like a neat way of establishing PROB, it has attracted (and continues to attract) more than its fair share of controversy...
- I can't review the entire massive literature here but I'll give you some key issues.
- I'll start with a recent contribution to the debate.

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3. Objections (1): 'Czech' books

- Alan Hajek [2005] points out that the threat of being Dutch booked comes hand in hand with an altogether more appealing prospect: the hope of being 'Czech' booked.
- What the *&#% is a Czech book?!?

Czech book: a set of bets on or against various propositions which, if accepted, would guarantee a net *gain*, whatever the truth values of the propositions with respect to which the bets in the set were made.
- It is indeed easy to establish the following:

CZECH: assuming BET, if Bel_S doesn't satisfy [P1] - [P4], then there exists a set of bets that *S* would consider fair which would collectively guarantee her a sure gain.

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3. Objections (1): ‘Czech’ books

- Furthermore, one can also easily prove:

CCZECH: assuming BET, if Bel_S does satisfy [P1] – [P4], then there *doesn't* exist a set of bets that S would consider fair which would collectively guarantee her a sure gain..
- Proof: take the Dutch books considered in the proofs for the Dutch and converse Dutch theorems and just multiply the stakes by -1 (if it isn't obvious to you that this works – although it should be – try it at home).
- So we have (here ‘coherent’ means ‘has d.o.b.s that obey the rules of probability’):

	◊ DB	◊ CB
Coherent	F ☹	F ☹
Incoherent	T ☺	T ☺

3. Objections (1): ‘Czech’ books

- Hajek takes this consideration to rob the DBA of its probative force: the advantage of avoiding DB's is offset by the disadvantage of missing out on CBs.
- He does however believe that the threat can be averted...
- Let us call a bet $B = \langle P, \mathcal{S}, q \rangle$ *favourable* for subject S iff (i) $q < \text{Bel}(P)$ if $\mathcal{S} > 0$ and (ii) $q > \text{Bel}(P)$ if $\mathcal{S} < 0$, where $\text{Bel}(P)$ is defined by BET.
- Example: say that my d.o.b in heads at the next coin toss is 0.5 ($\text{Bel}(H) = 0.5$). According to the definition just given, the following betting arrangements are favourable for me:
 - I pay you 25 pence ($-q\mathcal{S} = -0.25$) in return for receiving a pound if heads ($\mathcal{S} = 1$) and nothing if not (0). Here $\mathcal{S} > 0$ and $q = 0.25 < \text{Bel}(H)$.

3. Objections (1): ‘Czech’ books

- I take 75 pence of you ($-q\mathcal{S} = 0.75$) in return for giving you 1 pound if heads ($\mathcal{S} = -1$) and 0 pounds if not (0). $\mathcal{S} > 0$ and $q = 0.75 > \text{Bel}(H)$.
- As Hajek points out, the three following theorems are true:

DUTCH*: assuming BET, if Bel_S doesn't satisfy [P1] – [P4], then there exists a set of bets that S would consider *fair or favourable*, which would collectively guarantee her a sure loss.

Proof: this simply follows from DUTCH (if there exists a set of fair bets with those properties, there exists a set of fair or favourable bets with those properties). ■

3. Objections (1): ‘Czech’ books

CDUTCH*: assuming BET, if Bel_S *does* satisfy [P1] – [P4], then there *doesn't* exist a set of bets that S would consider fair or favourable, which would collectively guarantee her a sure loss.

Proof: see Hajek [2005:13] – the proof is really easy. ■

CZECH*: assuming BET, if Bel_S doesn't satisfy [P1] – [P4], then there exists a set of bets that S would consider fair or favourable, which would collectively guarantee her a sure gain.

Proof: simply follows from CZECH. ■

3. Objections (1): 'Czech' books

- The following putative theorem, however, is *false*:
 - **CCZECH***: assuming BET, if Bel_S *does* satisfy [P1] – [P4], then there *doesn't* exist a set of bets that S would consider fair or favourable, which would collectively guarantee her a sure gain.
- Proof: assuming BET, if Bel_S satisfies [P1] – [P4], then S would consider the following bet favourable: $\langle \Omega, 0.8, 1 \rangle$. This bet will guarantee a gain.
- So we have:

	◊ DB*	◊ CB*
Coherent	F ☹	T ☺
Incoherent	T ☹	T ☺

3. Objections (1): 'Czech' books

- Hajek takes this to restore the force of the argument: there is now a pragmatic asymmetry between being probabilistically coherent and being probabilistically incoherent.
- 'An important route to Bayesianism is clear again', he tells us.
- What do you reckon?

Reference

- Bartha, P. [2004]: 'Countable Additivity and the De Finetti Lottery', *BJPS* 55: 301-321.
- Hajek, A. [forthcoming]: 'Dutch Book Arguments', in P. Anand, P. Pattanaik, and C. Puppe (eds.) *The Oxford Handbook of Corporate Social Responsibility*.
- Hajek, A. [2005]: 'Scotching Dutch Books?', *Philosophical Perspectives* 19, *Epistemology*. Pp 139-151.
- Kemeny, J. [1955] 'Fair Bets and Inductive Probabilities', *Journal of Symbolic Logic*, 20: 263-273.
- Williamson, J. [1999]: 'Countable additivity and subjective probability', *British Journal for the Philosophy of Science* 50(3): 401-416.

Next lecture: 'More on Synchronic Dutch Book Arguments'

- Reading:
 - Christensen, D. [2004]: *Putting Logic in its Place*. Oxford: OUP. Pp 109-124.