Elements of Deductive Logic

Exercise set #2: Sentential Form and Truth Tables (solutions)

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1 Translation

Translate the following English arguments into the language of sentential (propositional) logic, providing the most descriptive sentential (propositional) form. Please provide the 'dictionaries' that you used for your translations.

(a) Tonight I'll go out or I'll stay at home. If I go out, I'll end up in a bar. If I stay at home, I'll get lonely. If I'm in a bar, I am very happy and I get drunk. If I'm lonely, I am very unhappy and I get drunk. Hence I will get drunk tonight.

$p \vee q$, $p \supset r$, $q \supset s$, $r \supset (t \& u)$, $s \supset (v \& u)$, therefore u.

p = I'll go out

q = I'll stay at home.

r = I'll end up in a bar.

s = I'll get lonely.

t = I'll be very happy

u = I'll get drunk.

v = I'll be very unhappy

(b) A person is entitled to state benefits only if either they are unemployed, or they are over 60 and they have a disposable income of less than £10, 000 per year. Therefore, if they have an income of less than £10, 000 per year and are over 60 they are entitled to state benefits.

$p \supset ((q \vee r) \& s)$ therefore $(r \& s) \supset p$ (Also acceptable: $p \supset (q \vee (r \& s))$ therefore $(r \& s) \supset p$)

p = They are entitled to state benefits

q = They are unemployed

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r = They are over 60

s = They have a disposable income of less than £10, 000 per year

(c) The blue tit has a blue head or it has a green head. If it has a green head, then the coal tit has a blue head or an orange head. The coal tit has neither of these. Therefore, the blue tit has a blue head.

$p \vee q$, $q \supset (r \vee s)$, $\sim r \& \sim s$, therefore p

p = The blue tit has a blue head

q = The blue tit has a green head

r = The coal tit has a blue head

s = The coal tit has an orange head

(d) If the Ukraine secedes from the treaty, and allies itself with Poland, then Georgia will ally itself with Russia. Georgia won't ally itself with the Baltic republics if the latter support economic decentralisation, and if Georgia allies itself with Russia, then the Baltic republics will support economic decentralisation or ask for help elsewhere. Therefore if the Ukraine secedes from the treaty and Georgia allies itself with the Baltic republics, then either the Baltic republics will ask for help elsewhere or the Ukraine won't ally itself with Poland.

$(p \& q) \supset r$, $(t \supset \sim s) \& (r \supset (t \vee u))$, therefore $(p \& s) \supset (u \vee \sim q)$

p = Ukraine secedes from the treaty

q = Ukraine allies itself with Poland

r = Georgia allies itself with Russia

s = Georgia allies itself with the Baltic republics

t = The Baltic republics support economic decentralisation

u = The Baltic republics ask for help elsewhere.

2 Truth tables

2.1 Tautologies and contradictions

Using the method of truth tables, determine of each of the following wfs's whether it is a tautology, a contradiction or a contingency:

(a) '$(\sim q \& (p \supset q)) \supset \sim p$' is a tautology:

p	q	($\sim$	q	$\&$	(p	$\supset$	q))	$\supset$	$\sim$	p
1	1	0	1	0	1	1	1	**1**	0	1
1	0	1	0	0	1	0	0	**1**	0	1
0	1	0	1	0	0	1	1	**1**	1	0
0	0	1	0	1	0	1	0	**1**	1	0

(b) ‘$((p \supset q)\&(\sim p \supset q)) \supset q$’ is a tautology:

p	q	$((p$	$\supset$	$q)$	$\&$	$(\sim$	p	$\supset$	$q))$	$\supset$	q
1	1	1	1	1	1	0	1	1	1	**1**	1
1	0	1	0	0	0	0	1	1	0	**1**	0
0	1	0	1	1	1	1	0	1	1	**1**	1
0	0	0	1	0	0	1	0	0	0	**1**	0

(c) ‘$\sim (p \vee q) \equiv (\sim p \& \sim q)$’ is a tautology:

p	q	$\sim$	$(p$	$\vee$	$q)$	$\equiv$	$(\sim$	p	$\&$	$\sim$	$q)$
1	1	0	1	1	1	**1**	0	1	0	0	1
1	0	0	1	1	0	**1**	0	1	0	1	0
0	1	0	0	1	1	**1**	1	0	0	0	1
0	0	1	0	0	0	**1**	1	0	1	1	0

(d) ‘$(p \equiv (q \equiv r)) \equiv ((p \equiv q) \equiv r))$’ is a tautology:

p	q	r	$(p$	$\equiv$	$(q$	$\equiv$	$r))$	$\equiv$	$((p$	$\equiv$	$q)$	$\equiv$	$r))$
1	1	1	1	1	1	1	1	**1**	1	1	1	1	1
1	1	0	1	0	1	0	0	**1**	1	1	1	0	0
1	0	1	1	0	0	0	1	**1**	1	0	0	0	1
1	0	0	1	1	0	1	0	**1**	1	0	0	1	0
0	1	1	0	0	1	1	1	**1**	0	0	1	0	1
0	1	0	0	1	1	0	0	**1**	0	0	1	1	0
0	0	1	0	1	0	0	1	**1**	0	1	0	1	1
0	0	0	0	0	0	1	0	**1**	0	1	0	0	0

2.2 Validity

Use the truth table method to check for validity. For each invalid argument, if any, indicate a truth assignment (valuation) that makes its premises true and conclusion false.

(a) ‘$p \supset (q \supset r)$, therefore $q \supset (p \supset r)$’ is valid:

p	q	r	p	$\supset$	$(q$	$\supset$	$r)$	q	$\supset$	$(p$	$\supset$	$r)$
1	1	1	1	**1**	1	1	1	1	**1**	1	1	1
1	1	0	1	**0**	1	0	0	1	**0**	1	0	0
1	0	1	1	**1**	0	1	1	0	**1**	1	1	1
1	0	0	1	**1**	0	1	0	0	**1**	1	0	0
0	1	1	0	**1**	1	1	1	1	**1**	0	1	1
0	1	0	0	**1**	1	0	0	1	**1**	0	1	0
0	0	1	0	**1**	0	1	1	0	**1**	0	1	1
0	0	0	0	**1**	0	1	0	0	**1**	0	1	0

(b) '$p \supset (q \vee r), \sim r$, therefore $\sim p$' is invalid. Countermodel in row 2 below: $v(p) = 1$, $v(q) = 1$ and $v(r) = 0$:

p	q	r	p	$\supset$	$(q$	$\vee$	$r)$	$\sim$	r	$\sim$	p
1	1	1	1	**1**	1	1	1	**0**	1	**0**	1
1	1	0	1	**1**	1	1	0	**1**	0	**0**	1
1	0	1	1	**1**	0	1	1	**0**	1	**0**	1
1	0	0	1	**0**	0	0	0	**1**	0	**0**	1
0	1	1	0	**1**	1	1	1	**0**	1	**1**	0
0	1	0	0	**1**	1	1	0	**1**	0	**1**	0
0	0	1	0	**1**	0	1	1	**0**	1	**1**	0
0	0	0	0	**1**	0	0	0	**1**	0	**1**	0

(c) '$(p \& q) \equiv r, p \equiv \sim q$, therefore $\sim r$' is valid:

p	q	r	$(p$	$\&$	$q)$	$\equiv$	r	p	$\equiv$	$\sim$	q	$\sim$	r
1	1	1	1	1	1	**1**	1	1	**0**	0	1	**0**	1
1	1	0	1	1	1	**0**	0	1	**0**	0	1	**1**	0
1	0	1	1	0	0	**0**	1	1	**1**	1	0	**0**	1
1	0	0	1	0	0	**1**	0	1	**1**	1	0	**1**	0
0	1	1	0	0	1	**0**	1	0	**1**	0	1	**0**	1
0	1	0	0	0	1	**1**	0	0	**1**	0	1	**1**	0
0	0	1	0	0	0	**0**	1	0	**0**	1	0	**0**	1
0	0	0	0	0	0	**1**	0	0	**0**	1	0	**1**	0