

Elements of Deductive Logic

Exercise set #4: Tableaux for many-valued logics

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1 Tableaux for K3 and LP

Check for validity in K3 and LP, providing countermodels if necessary:

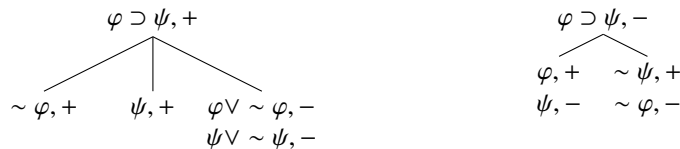
1. $p \& (q \vee r) \vdash (p \& q) \vee (p \& r)$
2. $(p \& q) \supset r \vdash (p \& \sim r) \supset \sim q$
3. $p \& \sim p \vdash q$
4. $p, \sim (p \& \sim q) \vdash q$
5. $(p \& q) \supset r \vdash p \supset (\sim q \vee r)$

2 The logic Ł3

The logic Ł3 is obtained from K3 by making the following amendment to the table for \supset , so that the equivalence between $\sim p \vee q$ and $p \supset q$ fails:

f_{\supset}	1	0	i
1	1	0	i
0	1	1	1
i	1	i	1

As is the case for K3, we have $V = \{0, i, 1\}$ and $D = \{1\}$. The rules for closing branches and generating countermodels remain the same. The pair of rules for \supset are as follows:



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$$\begin{array}{c} \sim(\varphi \supset \psi), + \\ | \\ \varphi, + \\ \sim\psi, + \end{array}$$

$$\begin{array}{c} \sim(\varphi \supset \psi), - \\ \swarrow \quad \searrow \\ \varphi, - \quad \sim\psi, - \end{array}$$

1. Justify the first rule (i.e. the rule for $\varphi \supset \psi, +$) with the use of truth tables.
2. The following are all classical tautologies. Use the tableau rules to establish which of them are also Ł3 tautologies, providing countermodels if and when appropriate.
 - (a) $\sim(p \& \sim p)$
 - (b) $p \supset ((p \supset q) \supset q)$
 - (c) $(p \supset \sim p) \supset \sim p$