Elements of Deductive Logic

Exercise set #5: Translation from and into Predicate Logic (solutions)

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Note: these solutions are *not* exhaustive; there may be satisfactory alternatives .

1 To $\mathcal{L}_P$

1.1 Single quantifiers

1. A snake is a reptile.

   $(\forall x)(Sx \supset Rx)$

2. A snake slipped into the kitchen.

   $(\exists x)(Sx \& Kx)$

3. Any snake found in the kitchen will be served for breakfast.

   $(\forall x)((Sx \& Fx) \supset Bx)$

4. If any snake is found in the kitchen then John won't be coming back.

   $(\exists x)(Sx \& Fx) \supset \sim Cj$
   (or equivalently $(\forall x)((Sx \& Fx) \supset \sim Cj)$)

5. Snakes are not all poisonous.

   $(\exists x)(Sx \& \sim Px)$

6. Children are present.

   $(\exists x)(Cx \& Px)$
   (and not $(\forall x)(Cx \supset Px)$!)

7. Executives all have secretaries.

   $(\forall x)(Ex \supset Sx)$

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8. Only executives have secretaries.

   $(\forall x)(Sx \supset Ex)$

9. Employees use only the service elevator.

   $(\forall x)(Ux \supset Sx)$
   ($U = \ldots$ is an elevator used by employees; $S = \ldots$ is a service elevator)

10. Only employees use the service elevator.

    $(\forall x)(Ux \supset Ex)$
    ($U = \ldots$ uses the elevator; $E = \ldots$ is an employee)

11. Not every visitor stayed for dinner.

    $(\exists x)(Vx \& \sim Sx)$

12. No coat is waterproof unless it has been specially treated.

    $(\forall x)(Cx \supset (\sim Wx \vee Sx))$

13. All fruits and vegetables are wholesome and delicious.

    $(\forall x)((Fx \vee Vx) \supset (Wx \& Dx))$

14. Only policemen and firemen are both indispensable and underpaid.

    $(\forall x)((Ix \& Ux) \supset (Px \vee Fx))$

15. If any bananas are yellow, they are ripe.

    $(\forall x)((Bx \& Yx) \supset Rx)$

1.2 Multiple quantifiers

1. If anything is damaged, someone will be blamed.

   $(\forall x)(Dx \supset (\exists y)(Py \& By))$

2. If nothing is damaged, nobody will be blamed.

   $\sim (\exists x)Dx \supset \sim (\exists y)(Py \& By)$
   (or again $\sim (\exists x)Dx \supset (\forall y)(Py \supset \sim By)$)

3. If all officers present are either captains or majors, then either some captain is present or some major is present.

   $(\forall x)((Ox \& Px) \supset (Cx \vee Mx)) \supset (\exists y)(Py \& (Cy \vee My))$

4. If any officer is present, then either no majors are present or he is a major.

   $(\forall x)((Ox \& Px) \supset (\sim (\exists y)(Py \& My) \vee Mx))$

5. If at least one officer is present, then if all officers present are captains, then some captains are present.

   $(\exists x)(Ox\&Px) \supset ((\forall y)((Oy\&Py) \supset Cy) \supset (\exists z)(Cz\&Pz))$

6. If all survivors are fortunate and only women were survivors, then if there are any survivors, then some women are fortunate.

   $(\forall x)((Sx \supset Fx)\&(Sx \supset Wx)) \supset ((\exists y)Sy \supset (\exists z)(Wz\&Fz))$

2 From $\mathcal{L}_P$

Dictionary:

G = . . . is a geologist

H = . . . is a hairdresser

P = . . . is a person

L = . . . is larger than. . .

Translate into English:

1. $(\forall x)(Gx \supset Hx)$

   All geologists are hairdressers.

2. $\sim (\forall x)(Gx \supset Hx)$

   Not all geologists are hairdressers (or: some geologists are not hairdressers)

3. $(\forall x)(Gx \supset\sim Hx)$

   No geologists are hairdressers.

4. $(\exists x)(Hx\&Px)$

   Someone is a hairdresser.

5. $(\forall x)(Px \supset (\exists y)(Py\&Lyx))$

   Everybody is smaller than someone.

6. $(\exists y)(Py\&(\forall x)(Px \supset Lyx))$

   There is someone who is larger than anyone (including him/herself!).

7. $(\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$

   If something is larger than a second thing, and that second thing is larger than a third, then the first thing is larger than the third.

8. $(\forall x)(\forall y)(Lxy \supset\sim Lyx)$

No two things are larger than each other.

9. $(\exists x)(Hx \& (\forall y)(Gy \supset Lxy))$

   There is a hairdresser who is larger than any geologist.

10. $(\exists x)(\forall y)(Gy \supset Lyx)$

    There exists something that is smaller than any geologist.