

ELEMENTS OF DEDUCTIVE LOGIC

16 & 17. Predicate Logic: tableau methods (ctd.) + identity + definite descriptions

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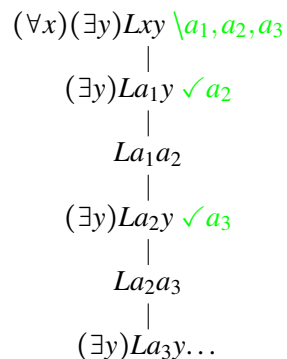
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Introduction

- Last time: tableaux for predicate logic
- This time:
 - finishing off tableaux
 - identity
 - definite descriptions

An interminable open tableau (!) (ctd.)

- Last time: an infinitely long completed open tableau:



- What about the countermodel?

An interminable open tableau (!) (ctd.)

- Ok, well our non-terminating branch will contain $La_1a_2, La_2a_3, La_3a_4, \dots$
- This gives us an *infinite* countermodel, which we present as follows, with $D = \{d_1, d_2, \dots\}$:

I		$I(L)$	d_1	d_2	d_3	d_4	\dots
a_1	d_1	d_1		1			\dots
a_2	d_2	d_2			1		\dots
a_3	d_3	d_3				1	\dots
a_4	d_4	d_4					\dots
\dots	\dots	\dots	\dots	\dots	\dots	\dots	\dots

- Note: the gaps denote entries for which you can choose any value you wish to.

An interminable open tableau (!) (ctd.)

- As Restall notes (p. 158), in this particular case, we can also use our initiative to find a *finite* countermodel.
- We simply have, where $D = \{d\}$:

$$\frac{I}{a \mid d}$$

$$\frac{I(L) \mid d}{d \mid 1}$$

- This is *not* always possible however.
- For instance, one can show that there is *no* finite model in which the following is true:

$$(\forall x)(\exists y)Rxy \& (\forall x) \sim Rxx \& (\forall x)(\forall y)(\forall z)((Rxy \& Ryz) \supset Rxz)$$

(Can you see why this might intuitively be the case?)

Some properties of quantifiers

- We finish off with some noteworthy validities and invalidities.
- Quantifier order:

$$\vdash (\forall x)(\forall y)A \equiv (\forall y)(\forall x)A$$

$$\vdash (\exists x)(\exists y)A \equiv (\exists y)(\exists x)A$$

$$\text{But } \not\vdash (\forall y)(\exists x)A \equiv (\exists x)(\forall y)A$$

$(\forall y)(\exists x)Lxy$: Everyone is loved by someone or other.

$(\exists x)(\forall y)Lxy$: There is someone who loves everyone.

- Quantifier duality:

$$\vdash (\forall x)A \equiv \sim (\exists x) \sim A$$

$$\vdash (\exists x)A \equiv \sim (\forall x) \sim A$$

Some properties of quantifiers

- Finally, we have the following:
 $\sim (\exists x)Fx \vdash (\forall x)(Fx \supset Gx)$
- So we can infer from the fact that there are no flying turtles that all flying turtles fly South in the Winter.
- In L8, we saw that this is somewhat controversial.
- In particular, Strawson would say that when the premise is true, which it can be, the conclusion is neither true nor false.
- So on our trivalent logics for reasoning about gaps, the inference would be invalid (1 is designated but *i* is not)

Reasoning with identity

- An apparently valid inference:
Superman is threatening to renounce his US citizenship.*
Superman is Clark Kent.
Therefore Clark Kent is threatening to renounce his US citizenship.
- * Reported @ <http://scans-daily.dreamwidth.org/2939770.html>
- In standard predicate logic:
 - Premises: R_s, I_{sc}
 - Conclusion: R_c
- Problem: $R_s, I_{sc} \not\equiv R_c$
- Perhaps we have the hidden premise $R_s \supset R_c$?
- This seems implausible: it looks like this *follows* from I_{sc} , not that it needs to be independently postulated.

Reasoning with some new quantifiers

- Another apparently valid inference:
John has three problems.
Therefore John has at least two problems.
- What is the logical form?
- Suggestion:
 - Premise: $(\exists x)(Px \& H jx) \& (\exists y)(Py \& H jy) \& (\exists z)(Pz \& H jz)$
 - Conclusion: $(\exists x)(Px \& H jx) \& (\exists y)(Py \& H jy) \vee \dots$
- Problem: $(\exists x)(Px \& H jx) \& (\exists y)(Py \& H jy) \& (\exists z)(Pz \& H jz)$ is equivalent to $(\exists x)(Px \& H jx)$.
- We could use two unary predicates: ‘... has three problems’ (F) and ‘... has at least two problems’ (G).
- Problem: $F j \neq G j$; same issue as previous slide.

Expanding the language: syntax and semantics

- We are going to add a special symbol ‘=’ to \mathcal{L}_P to yield \mathcal{L}_{PI} (for ‘predicate logic with identity’)
- New syntactic rule:
If a and b are names, then $a = b$ is a wfs.
- New semantic rule:
Where a and b are names, $v_M(a = b) = 1$ iff $I(a) = I(b)$.
- We write ‘ $a \neq b$ ’ as shorthand for $\sim a = b$

Amending the tableau system

- New rule for closure:
For any name a , if $a \neq a$ is on a branch b , then b closes.
- New tableau rule, where φ is a wfs, a and b are names and $\varphi(a := b)$ denotes the result of substituting b for the instances of a in φ :

$$\begin{array}{c} \varphi \\ a = b \\ | \\ \varphi(a := b) \end{array}$$

- We can now translate the Superman argument: $R_s, s = c$ therefore R_c . Proof of validity left as a (trivial) exercise.
- More on the second case shortly.

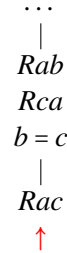
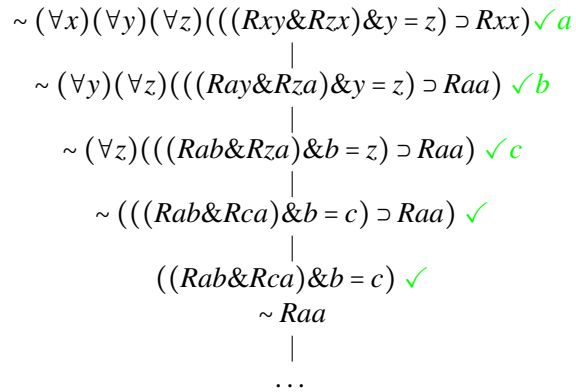
A tableau that closes

- We show that $\vdash (\forall x)(\forall y)(x = y \supset y = x)$:

$$\begin{array}{l} \sim (\forall x)(\forall y)(x = y \supset y = x) \quad \checkmark a \\ | \\ \sim (\forall y)(a = y \supset y = a) \quad \checkmark b \\ | \\ \sim (a = b \supset b = a) \quad \checkmark \\ | \\ a = b \\ b \neq a \\ | \\ b \neq b \\ \times \end{array}$$

A completed open tableau

- We show that $\not\vdash (\forall x)(\forall y)(\forall z)((Rxy \& Rzx) \& y = z) \supset Rxx$



A completed open tableau

- What about the countermodel?
- Same as before *except* that identity sentences on the relevant open branch constrain the interpretation of names.
- Here, we must have $I(b) = I(c)$.
- Note: on p. 172 Restall writes: ‘we may not always take the domain constructed from the model to have just one object per name’. He means ‘name per object’.
- Our countermodel: $D = \{d, e\}$, with

I	
a	d
b	e
c	e

$I(R)$	d	e
d	0	1
e		1

Handling our new quantifiers

- Ok, back to the second argument:
John has three problems.
Therefore John has at least two problems.
- Regarding the second premise:
 $(\exists x)(\exists y)(Pxj \& Pyj \& x \neq y)$
‘There exists an x , such that there exists a y , such that x and y are both problems of John’s and are non-identical.’

Handling our new quantifiers (ctd.)

- More generally, we translate ‘There are at least n F ’s’ by
 $(\exists x_1) \dots (\exists x_n)(Fx_1 \& \dots \& Fx_n \& x_1 \neq x_2 \& \dots \& x_1 \neq x_n \& \dots \& x_2 \neq x_3 \& \dots \& x_2 \neq x_n \& \dots \& x_{n-1} \neq x_n)$
‘There exists an x_1 , such that there exists an x_2, \dots , such that there exists an x_n , such that x_1 to x_n are all F ’s and are non-identical.’
- We can also translate ‘There are at most n F ’s’:
 $(\forall x_1) \dots (\forall x_{n+1})((Fx_1 \& \dots \& Fx_{n+1}) \supset (x_1 = x_2 \vee \dots \vee x_1 = x_n \vee \dots \vee x_2 = x_3 \vee \dots \vee x_2 = x_n \vee \dots \vee x_n = x_{n+1}))$
‘For all x_1 to x_{n+1} , if x_1 to x_{n+1} are all F ’s, then at least two of them are identical.’

Handling our new quantifiers (ctd.)

- With this we can now translate ‘There are (exactly) n F ’s’ via its equivalence with ‘There are at least and at most n F ’s’.
- So, for ‘John has exactly two problems’, we get:

$$(\exists x)(\exists y)(Pxj \& Pyj \& x \neq y) \& (\forall x)(\forall y)(\forall z)((Pxj \& Pyj \& Pzj) \supset (x = y \vee x = z \vee y = z))$$
- Or more compactly:

$$(\exists x)(\exists y)(Pxj \& Pyj \& x \neq y \& (\forall z)(Pzj \supset (x = z \vee y = z)))$$

‘There exists an x , such that there exists a y , such that x and y are both problems of John’s and are non-identical *and* such that for all z , if z is a problem of John’s then it is identical to either x or y .’
- These new quantifiers turn out to play a central role in discussions of definite descriptions...

Introduction

- The logic of definite descriptions (mentioned in L8) is a major topic in philosophy of language.
- We won’t be able to do justice to it here. Good overviews: Ludlow (2007) and Ludlow & Neale (2006).
- Examples:
 - The last one to wake up* gets to do the washing up.
 - The groom* was wearing a ridiculous outfit.
- Note: these are *singular* definite descriptions; we leave aside *plural* forms, e.g.
 - The cats* were sleeping again.

Introduction (ctd.)

- We also set aside *indefinite* descriptions, both singular and plural:
 - A man* was fishing a few yards away.
 - Men* were fishing a few yards away.
- Finally, we are not discussing so called ‘generics’:
 - The tiger* is an endangered species.

Plausibly, these are simply universally quantified conditionals.
- Question: how do we translate (1) and (2) into \mathcal{L}_P ?
- Could we use constants (i.e. names)? Are definite descriptions designators like proper names are?

Introduction (ctd.)

- If they are designators *and* we have gotten the semantics of denoting expressions right, there is trouble...
- Either $v_M(Fa) = 1$ or $v_M(\sim Fa) = 1$, so one or the other of the following would have to be true:
 - ‘The present king of France is bald.’
 - ‘The present king of France is not bald.’

But intuitively, neither of them are.
- This seems to be because of a violation of a requirement that there exists something that satisfies the description.

Russell's proposal

- Russell (1905): satisfaction of this existence condition (+ a uniqueness condition) is part of the logical form of (7) & (8)...
- His suggestion: we translate (7) by 'There exists exactly one King of France and that person is bald'.
- In our language \mathcal{L}_P :

$$(\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& Bx)$$
- Or equivalently:

$$(\exists x)(Kx \& (\forall y)(Ky \supset y = x)) \& (\forall x)(Kx \supset Bx)$$
- Regarding (8), we add a negation inside the scope of the main quantifier:

$$(\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& \sim Bx)$$

Russell's proposal

- The translation for (8) is *not* the negation of the translation for (7)
- *That* would be

$$\sim (\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& Bx)$$
- So no failure of bivalence.

A syntactic and proof-theoretic shortcut

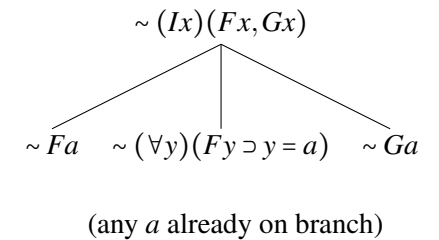
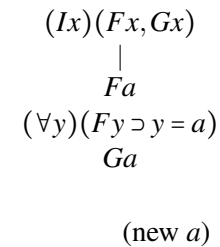
- The Russelian translation yields something very cumbersome
- Some new notation:
 We write $(Ix)(Fx, Gx)$ for $(\exists x)(Fx \& (\forall y)(Fy \supset y = x) \& Gx)$
- We can rewrite our translation of (7) as $(Ix)(Kx, Bx)$.
- This generalises for sentences with multiple definite descriptions:
 (9) 'The turkey ran after the goose'.
- Longhand translation:

$$(\exists x)(Tx \& (\forall y)(Ty \supset y = x) \& (\exists y)(Gy \& (\forall z)(Gz \supset z = y) \& Rxy))$$
- Shorthand translation:

$$(Ix)(Tx, (Iy)(Gy, Rxy))$$

Extra tableau rules

- Restall (p. 187) gives us a pair of fairly obvious corresponding tableau rules, one particular, one general:



- Note: he forgets the negation in the middle branch of the second rule.

A trivial example

- We show that $(Ix)(Fx, Gx) \vdash (\exists x)Fx$:

$$\begin{array}{c}
 (Ix)(Fx, Gx) \checkmark a \\
 \sim (\exists x)Fx \setminus a \\
 | \\
 Fa \\
 (\forall y)(Fy \supset y = a) \\
 Ga \\
 | \\
 \sim Fa \\
 \times
 \end{array}$$

Problems for Russell (2): pronominal anaphora

- Strawson (1952) brings attention to cases like:
 - (12) *The officer on duty* was in a bad mood. *He* had been dealing with claims all morning.
- The argument is then: since ‘he’ is a designator, so too is the expression that it is anaphoric on, namely ‘the officer on duty’.
- One response: ‘the officer on duty’ is not a designator, so neither is the corresponding anaphoric pronoun ‘he’.

Problems for Russell (1): intuitions about gappiness

- Russell’s view is *very* popular, but it faces some difficulties.
- As we have seen, some people (e.g. Strawson 1950),
 - agree that both (7) and (8) are untrue, but
 - hold that they are so because they are *neither true nor false*.
- In response: cases in which intuitions allegedly clearly go against Strawson’s (Neale 1990; but see von Fintel 2004):
 - (10) ‘This morning, I had tea with the king of France.’
- What do you reckon?

Thank you, and good luck with the exam!

References

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