ELEMENTS OF DEDUCTIVE LOGIC

16 & 17. Predicate Logic: tableau methods (ctd.) + identity + definite descriptions

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Introduction

- Last time: tableaux for predicate logic
- This time:
  - finishing off tableaux
  - identity
  - definite descriptions

An interminable open tableau (!) (ctd.)

- Last time: an infinitely long completed open tableau:

$$(\forall x)(\exists y)Lxy \setminus a_1, a_2, a_3$$
$$|$$
$$(\exists y)La_1y \checkmark a_2$$
$$|$$
$$La_1a_2$$
$$|$$
$$(\exists y)La_2y \checkmark a_3$$
$$|$$
$$La_2a_3$$
$$|$$
$$(\exists y)La_3y\ldots$$

- What about the countermodel?

An interminable open tableau (!) (ctd.)

- Ok, well our non-terminating branch will contain $La_1a_2, La_2a_3, La_3a_4, \ldots$
- This gives us an *infinite* countermodel, which we present as follows, with $D = \{d_1.d_2,\ldots\}$:

I	
a_1	d_1
a_2	d_2
a_3	d_3
a_4	d_4
...	...

$I(L)$	d_1	d_2	d_3	d_4	...
d_1		1			...
d_2			1		...
d_3				1	...
d_4					...
...	...	...	...	...	...

- Note: the gaps denote entries for which you can choose any value you wish to.

An interminable open tableau (!) (ctd.)

- As Restall notes (p. 158), in this particular case, we can also use our initiative to find a *finite* countermodel.
- We simply have, where $D = \{d\}$:

I	
a	d

$I(L)$	d
d	1

- This is *not* always possible however.
- For instance, one can show that there is *no* finite model in which the following is true:
  $(\forall x)(\exists y)Rxy \& (\forall x) \sim Rxx \& (\forall x)(\forall y)(\forall z)((Rxy \& Ryz) \supset Rxz)$
  (Can you see why this might intuitively be the case?)

Some properties of quantifiers

- We finish off with some noteworthy validities and invalidities.
- Quantifier order:
  $\vdash (\forall x)(\forall y)A \equiv (\forall y)(\forall x)A$
  $\vdash (\exists x)(\exists y)A \equiv (\exists y)(\exists x)A$
  But $\nvdash (\forall y)(\exists x)A \equiv (\exists x)(\forall y)A$
  $(\forall y)(\exists x)Lxy$: Everyone is loved by someone or other.
  $(\exists x)(\forall y)Lxy$: There is someone who loves everyone.
- Quantifier duality:
  $\vdash (\forall x)A \equiv \sim (\exists x) \sim A$
  $\vdash (\exists x)A \equiv \sim (\forall x) \sim A$

Some properties of quantifiers

- Finally, we have the following:
  $\sim (\exists x)Fx \vdash (\forall x)(Fx \supset Gx)$
- So we can infer from the fact that there are no flying turtles that all flying turtles fly South in the Winter.
- In L8, we saw that this is somewhat controversial.
- In particular, Strawson would say that when the premise is true, which it can be, the conclusion is neither true nor false.
- So on our trivalent logics for reasoning about gaps, the inference would be invalid (1 is designated but i is not)

Reasoning with identity

- An apparently valid inference:
  Superman is threatening to renounce his US citizenship.*
  Superman is Clark Kent.
  Therefore Clark Kent is threatening to renounce his US citizenship.

  * Reported @ http://scans-daily.dreamwidth.org/2939770.html
- In standard predicate logic:
  - Premises: Rs, Isc
  - Conclusion: Rc
- Problem: $Rs, Isc \nvDash Rc$
- Perhaps we have the hidden premise $Rs \supset Rc$?
- This seems implausible: it looks like this *follows* from Isc, not that itneeds to be independently postulated.

Reasoning with some new quantifiers

- Another apparently valid inference:

  John has three problems.
  Therefore John has at least two problems.
- What is the logical form?
- Suggestion:
  - Premise: $(\exists x)(Px \& Hjx) \& (\exists y)(Py \& Hjy) \& (\exists z)(Pz \& Hjz)$
  - Conclusion: $(\exists x)(Px \& Hjx) \& (\exists y)(Py \& Hjy) \vee \ldots$
- Problem: $(\exists x)(Px \& Hjx) \& (\exists y)(Py \& Hjy) \& (\exists z)(Pz \& Hjz)$ is equivalent to $(\exists x)(Px \& Hjx)$.
- We could use two unary predicates: '…has three problems' (F) and '…has at least two problems' (G).
- Problem: $Fj \nvDash Gj$; same issue as previous slide.

Expanding the language: syntax and semantics

- We are going to add a special symbol '$=$' to $\mathcal{L}_P$ to yield $\mathcal{L}_{PI}$ (for 'predicate logic with identity')
- New syntactic rule:

  If a and b are names, then $a = b$ is a wfs.
- New semantic rule:

  Where a and b are names, $v_M(a = b) = 1$ iff $I(a) = I(b)$.
- We write '$a \neq b$' as shorthand for $\sim a = b$

Amending the tableau system

- New rule for closure:

  For any name a, if $a \neq a$ is on a branch b, then b closes.
- New tableau rule, where φ is a wfs, a and b are names and $\varphi(a := b)$ denotes the result of substituting b for the instances of a in φ:

$$\begin{array}{c} \varphi \\ a = b \\ | \\ \varphi(a := b) \end{array}$$

- We can now translate the Superman argument: $Rs, s = c$ therefore Rc. Proof of validity left as a (trivial) exercise.
- More on the second case shortly.

A tableau that closes

- We show that $\vdash (\forall x)(\forall y)(x = y \supset y = x)$:

$$\begin{array}{c} \sim (\forall x)(\forall y)(x = y \supset y = x) \ \checkmark a \\ | \\ \sim (\forall y)(a = y \supset y = a) \ \checkmark b \\ | \\ \sim (a = b \supset b = a) \ \checkmark \\ | \\ a = b \\ b \neq a \\ | \\ b \neq b \\ \times \end{array}$$

A completed open tableau

- We show that $\nvDash (\forall x)(\forall y)(\forall z)(((Rxy \& Rzx) \& y = z) \supset Rxx)$

$\sim (\forall x)(\forall y)(\forall z)(((Rxy \& Rzx) \& y = z) \supset Rxx) \checkmark a$

|

$\sim (\forall y)(\forall z)(((Ray \& Rza) \& y = z) \supset Raa) \checkmark b$

|

$\sim (\forall z)(((Rab \& Rza) \& b = z) \supset Raa) \checkmark c$

|

$\sim (((Rab \& Rca) \& b = c) \supset Raa) \checkmark$

|

$((Rab \& Rca) \& b = c) \checkmark$

$\sim Raa$

|

$\ldots$

$\ldots$

|

Rab

Rca

$b = c$

|

Rac

$\uparrow$

A completed open tableau

- What about the countermodel?
- Same as before *except* that identity sentences on the relevant open branch constrain the interpretation of names.
- Here, we must have $I(b) = I(c)$.
- Note: on p. 172 Restall writes:‘we may not always take the domain constructed from the model to have just one object per name’. He means ‘*name* per *object*’.
- Our countermodel: $D = \{d, e\}$, with

I	
a	d
b	e
c	e

$I(R)$	d	e
d	0	1
e	1	

Handling our new quantifiers

- Ok, back to the second argument:

  John has three problems.
  Therefore John has at least two problems.
- Regarding the second premise:

  $(\exists x)(\exists y)(Pxj \& Pyj \& x \neq y)$

  ‘There exists an x, such that there exists a y, such that x and y are both problems of John’s and are non-identical.’

Handling our new quantifiers (ctd.)

- More generally, we translate ‘There are at least n F’s’ by

  $(\exists x_1)\ldots(\exists x_n)(Fx_1 \& \ldots \& Fx_n \& x_1 \neq x_2 \& \ldots \& x_1 \neq x_n \& \ldots \& x_2 \neq x_3 \& \ldots \& x_2 \neq x_n \& \ldots \& x_{n-1} \neq x_n)$

  ‘There exists an x_1, such that there exists an x_2, …, such that there exists an x_n, such that x_1 to x_n are all F’s and are non-identical.’
- We can also translate ‘There are at most n F’s’:

  $(\forall x_1)\ldots(\forall x_{n+1})((Fx_1 \& \ldots \& Fx_{n+1}) \supset (x_1 = x_2 \vee \ldots \vee x_1 = x_n \vee \ldots \vee x_2 = x_3 \vee \ldots \vee x_2 = x_n \vee \ldots \vee x_n = x_{n+1}))$

  ‘For all x_1 to x_{n+1}, if x_1 to x_{n+1} are all F’s, then at least two of them are identical.’

Handling our new quantifiers (ctd.)

- With this we can now translate 'There are (exactly) *n F*'s' via its equivalence with 'There are at least and at most *n F*'s'.
- So, for 'John has exactly two problems', we get:
  $(\exists x)(\exists y)(Pxj \& Pyj \& x \neq y) \& (\forall x)(\forall y)(\forall z)((Pxj \& Pyj \& Pzj) \supset (x = y \vee x = z \vee y = z))$
- Or more compactly:
  $(\exists x)(\exists y)(Pxj \& Pyj \& x \neq y \& (\forall z)(Pzj \supset (x = z \vee y = z)))$

  'There exists an x, such that there exists a y, such that x and y are both problems of John's and are non-identical *and* such that for all z, if z is a problem of John's then it is identical to either x or y.'
- These new quantifiers turn out to play a central role in discussions of definite descriptions...

Introduction

- The logic of definite desciptions (mentioned in L8) is a major topic in philosophy of language.
- We won't be able to do justice to it here. Good overviews: Ludlow (2007) and Ludlow & Neale (2006).
- Examples:
  (1) *The last one to wake up* gets to do the washing up.
  (2) *The groom* was wearing a ridiculous outfit.
- Note: these are *singular* definite descriptions; we leave aside *plural* forms, e.g.
  (3) *The cats* were sleeping again.

Introduction (ctd.)

- We also set aside *indefinite* descriptions, both singular and plural:
  (4) *A man* was fishing a few yards away.
  (5) *Men* were fishing a few yards away.
- Finally, we are not discussing so called 'generics':
  (6) *The tiger* is an endangered species.

  Plausibly, these are simply universally quantified conditionals.
- Question: how do we translate (1) and (2) into $\mathcal{L}_P$?
- Could we use constants (i.e. names)? Are definite descriptions designators like proper names are?

Introduction (ctd.)

- If they are designators *and* we have gotten the semantics of denoting expressions right, there is trouble...
- Either $v_M(Fa) = 1$ or $v_M(\sim Fa) = 1$, so one or the other of the following would have to be true:
  (7) 'The present king of France is bald.'
  (8) 'The present king of France is not bald.'

  But intuitively, neither of them are.
- This seems to be because of a violation of a requirement that there exists something that satisfies the description.

Russell's proposal

- Russell (1905): satisfaction of this existence condition (+ a uniqueness condition) is part of the logical form of (7) & (8)...
- His suggestion: we translate (7) by 'There exists exactly one King of France and that person is bald'.
- In our language $\mathcal{L}_P$:

  $(\exists x)(Kx\&(\forall y)(Ky \supset y = x)\&Bx)$
- Or equivalently:

  $(\exists x)(Kx\&(\forall y)(Ky \supset y = x))\&(\forall x)(Kx \supset Bx)$
- Regarding (8), we add a negation inside the scope of the main quantifier:

  $(\exists x)(Kx\&(\forall y)(Ky \supset y = x)\&{\sim}Bx)$

Russell's proposal

- The translation for (8) is *not* the negation of the translation for (7)
- *That* would be

  ${\sim}(\exists x)(Kx\&(\forall y)(Ky \supset y = x)\&Bx)$
- So no failure of bivalence.

A syntactic and proof-theoretic shortcut

- The Russelian translation yields something very cumbersome
- Some new notation:

  We write $(Ix)(Fx,Gx)$ for $(\exists x)(Fx\&(\forall y)(Fy \supset y = x)\&Gx)$
- We can rewrite our translation of (7) as $(Ix)(Kx,Bx)$.
- This generalises for sentences with multiple definite descriptions:

  (9) 'The turkey ran after the goose'.
- Longhand translation:

  $(\exists x)(Tx\&(\forall y)(Ty \supset y = x)\&(\exists y)(Gy\&(\forall z)(Gz \supset z = y)\&Rxy))$
- Shorthand translation:

  (Ix)(Tx, (Iy)(Gy, Rxy))

Extra tableau rules

- Restall (p. 187) gives us a pair of fairly obvious corresponding tableau rules, one particular, one general:

$(Ix)(Fx,Gx)$
|
Fa
$(\forall y)(Fy \supset y = a)$
Ga

(new *a*)

${\sim}(Ix)(Fx,Gx)$

${\sim}Fa$ | ${\sim}(\forall y)(Fy \supset y = a)$ | ${\sim}Ga$

(any *a* already on branch)

- Note: he forgets the negation in the middle branch of the second rule.

A trivial example

- We show that $(Ix)(Fx,Gx) \vdash (\exists x)Fx$:

$$
\begin{array}{c}
(Ix)(Fx,Gx) \checkmark a \\
\sim(\exists x)Fx \setminus a \\
| \\
Fa \\
(\forall y)(Fy \supset y = a) \\
Ga \\
| \\
\sim Fa \\
\times
\end{array}
$$

Problems for Russell (1): intuitions about gappiness

- Russell's view is *very* popular, but it faces some difficulties.
- As we have seen, some people (e.g. Strawson 1950),
  - agree that both (7) and (8) are untrue, but
  - hold that they are so because they are *neither true nor false*.
- In response: cases in which intuitions allegedly clearly go against Strawson's (Neale 1990; but see von Fintel 2004):

(10) 'This morning, I had tea with the king of France.'

- What do you reckon?

Problems for Russell (2): pronominal anaphora

- Strawson (1952) brings attention to cases like:

(12) *The officer on duty* was in a bad mood. *He* had been dealing with claims all morning.

- The argument is then: since 'he' is a designator, so too is the expression that it is anaphoric on, namely 'the officer on duty'.
- One response: 'the officer on duty' is not a designator, so neither is the corresponding anaphoric pronoun 'he'.

Thank you, and good luck with the exam!

References

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