ELEMENTS OF DEDUCTIVE LOGIC

16 & 17. Predicate Logic: tableau methods (ctd.) + identity + definite descriptions

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Introduction

- Last time: tableaux for predicate logic
- This time:
  - finishing off tableaux
  - identity
  - definite descriptions

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An interminable open tableau (!) (ctd.)

- Last time: an infinitely long completed open tableau:
- What about the countermodel?

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- Last time: an infinitely long completed open tableau:

$$(\forall x)(\exists y)Lxy \setminus a_1, a_2, a_3$$
$$|$$
$$(\exists y)La_1y \checkmark a_2$$
$$|$$
$$La_1a_2$$
$$|$$
$$(\exists y)La_2y \checkmark a_3$$
$$|$$
$$La_2a_3$$
$$|$$
$$(\exists y)La_3y\ldots$$

- What about the countermodel?

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- Ok, well our non-terminating branch will contain $La_1a_2, La_2a_3, La_3a_4, \ldots$
- This gives us an *infinite* countermodel, which we present as follows, with $D = \{d_1.d_2, \ldots\}$:

I	
a_1	d_1
a_2	d_2
a_3	d_3
a_4	d_4
$\ldots$	$\ldots$

$I(L)$	d_1	d_2	d_3	d_4	$\ldots$
d_1		1			$\ldots$
d_2			1		$\ldots$
d_3				1	$\ldots$
d_4					$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

- Note: the gaps denote entries for which you can choose any value you wish to.

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An interminable open tableau (!) (ctd.)

- As Restall notes (p. 158), in this particular case, we can also use our initiative to find a *finite* countermodel.
- We simply have, where $D = \{d\}$:

I	
a	d

$I(L)$	d
d	1

- This is *not* always possible however.
- For instance, one can show that there is *no* finite model in which the following is true:

$$(\forall x)(\exists y)Rxy \& (\forall x) \sim Rxx \& (\forall x)(\forall y)(\forall z)((Rxy \& Ryz) \supset Rxz)$$

(Can you see why this might intuitively be the case?)

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Some properties of quantifiers

- We finish off with some noteworthy validities and invalidities.
- Quantifier order:

  $\vdash (\forall x)(\forall y)A \equiv (\forall y)(\forall x)A$
  $\vdash (\exists x)(\exists y)A \equiv (\exists y)(\exists x)A$
  But $\nvdash (\forall y)(\exists x)A \equiv (\exists x)(\forall y)A$

  $(\forall y)(\exists x)Lxy$: Everyone is loved by someone or other.
  $(\exists x)(\forall y)Lxy$: There is someone who loves everyone.

- Quantifier duality:

  $\vdash (\forall x)A \equiv \sim (\exists x) \sim A$
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- Finally, we have the following:
  $\sim(\exists x)Fx \vdash (\forall x)(Fx \supset Gx)$
- So we can infer from the fact that there are no flying turtles that all flying turtles fly South in the Winter.
- In L8, we saw that this is somewhat controversial.
- In particular, Strawson would say that when the premise is true, which it can be, the conclusion is neither true nor false.
- So on our trivalent logics for reasoning about gaps, the inference would be invalid (1 is designated but *i* is not)

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Reasoning with identity

- An apparently valid inference:

  Superman is threatening to renounce his US citizenship.*
  Superman is Clark Kent.
  Therefore Clark Kent is threatening to renounce his US citizenship.

  * Reported @ http://scans-daily.dreamwidth.org/2939770.html
- In standard predicate logic:
  - Premises: Rs, Isc
  - Conclusion: Rc
- Problem: $Rs, Isc \not\models Rc$
- Perhaps we have the hidden premise $Rs \supset Rc$?
- This seems implausible: it looks like this *follows* from Isc, not that itneeds to be independently postulated.

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  Superman is Clark Kent.
  Therefore Clark Kent is threatening to renounce his US citizenship.

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  - Premises: Rs, Isc
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Reasoning with some new quantifiers

- Another apparently valid inference:

  John has three problems.
  Therefore John has at least two problems.

- What is the logical form?
- Suggestion:
  - Premise: $(\exists x)(Px\&Hjx)\&(\exists y)(Py\&Hjy)\&(\exists z)(Pz\&Hjz)$
  - Conclusion: $(\exists x)(Px\&Hjx)\&(\exists y)(Py\&Hjy) \vee \ldots$
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Expanding the language: syntax and semantics

- We are going to add a special symbol '=' to $\mathcal{L}_P$ to yield $\mathcal{L}_{PI}$ (for 'predicate logic with identity')
- New syntactic rule:

  If a and b are names, then $a = b$ is a wfs.
- New semantic rule:

  Where a and b are names, $v_M(a = b) = 1$ iff $I(a) = I(b)$.
- We write '$a \neq b$' as shorthand for $\sim a = b$

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Amending the tableau system

- New rule for closure:

  For any name a, if $a \neq a$ is on a branch b, then b closes.

- New tableau rule, where φ is a wfs, a and b are names and $\varphi(a := b)$ denotes the result of substituting b for the instances of a in φ:

- We can now translate the Superman argument: $Rs, s = c$ therefore Rc. Proof of validity left as a (trivial) exercise.
- More on the second case shortly.

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$$|$$

$$\sim (\forall y)(a = y \supset y = a)$$

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$$|$$

$$\sim (\forall y)(a = y \supset y = a) \ \checkmark b$$

$$|$$

$$\sim (a = b \supset b = a)$$

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$$|$$

$$\sim (\forall y)(a = y \supset y = a) \; \checkmark b$$

$$|$$

$$\sim (a = b \supset b = a) \; \checkmark$$

$$|$$

$$a = b$$

$$b \neq a$$

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$$|$$

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$$|$$

$$\sim (a = b \supset b = a) \ \checkmark$$

$$|$$

$$a = b$$

$$b \neq a$$

$$|$$

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$$\times$$

A completed open tableau

- We show that $\nvdash (\forall x)(\forall y)(\forall z)(((Rxy \& Rzx) \& y = z) \supset Rxx)$

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$$Rab$$

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$$|$$

$$Rac$$

$$\uparrow$$

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- What about the countermodel?
- Same as before *except* that identity sentences on the relevant open branch constrain the interpretation of names.
- Here, we must have $I(b) = I(c)$.
- Note: on p. 172 Restall writes:'we may not always take the domain constructed from the model to have just one object per name'. He means '*name* per *object*'.
- Our countermodel: $D = \{d, e\}$, with

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- Our countermodel: $D = \{d, e\}$, with

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b	e
c	e

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A completed open tableau

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  John has three problems.
  Therefore John has at least two problems.

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- More generally, we translate 'There are at least n F's' by

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- We won't be able to do justice to it here. Good overviews: Ludlow (2007) and Ludlow & Neale (2006).
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  (1) *The last one to wake up* gets to do the washing up.
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  - (4) *A man* was fishing a few yards away.
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  (7) 'The present king of France is bald.'
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Russell's proposal

- Russell (1905): satisfaction of this existence condition (+ a uniqueness condition) is part of the logical form of (7) & (8)…
- His suggestion: we translate (7) by 'There exists exactly one King of France and that person is bald'.
- In our language $\mathcal{L}_P$:

  $(\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& Bx)$
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- The Russelian translation yields something very cumbersome
- Some new notation:

  We write $(Ix)(Fx, Gx)$ for $(\exists x)(Fx\&(\forall y)(Fy \supset y = x)\&Gx)$

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- Restall (p. 187) gives us a pair of fairly obvious corresponding tableau rules, one particular, one general:

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\times
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- Russell's view is *very* popular, but it faces some difficulties.
- As we have seen, some people (e.g. Strawson 1950),
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- In response: cases in which intuitions allegedly clearly go against Strawson's (Neale 1990; but see von Fintel 2004):

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  (12) *The officer on duty* was in a bad mood. *He* had been dealing with claims all morning.

- The argument is then: since 'he' is a designator, so too is the expression that it is anaphoric on, namely 'the officer on duty'.
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Thank you, and good luck with the exam!

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