ELEMENTS OF DEDUCTIVE LOGIC

3. Sentential form

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KUL 2012

Sentences and sub-sentences

- Before we get stuck in to our main topic, we first need some terminology…
- A complex sentence is a sentence built up from other sentences.
- These sentential building blocks, which can themselves be complex, are known as sub-sentences.
- Not all sentences are complex, however.
- An atomic sentence is a sentence that has no further component sub-sentences.

Atomic vs complex sentences

Atomic vs complex

Atomic sentences:

It'll be crowded.

He stayed in Damascus.

The lights are on.

Compound sentences (sub-sentences in italics):

Either *it'll be crowded* or *it'll be empty*.

If *he stayed in Damascus*, then *he will have been arrested*.

It is not the case that *the lights are on*.

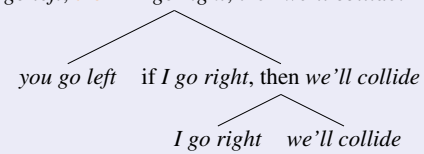
Connectives

- In the examples we have just seen, some parts of the complex sentences were not sub-sentences.
- These elements are known as sentential connectives.
- They take one or more sentences as an input and produce a complex sentence as an output.
- There are *many* connectives, but we will mainly focus on the following five:
  - …and…
  - …or…
  - if…, then…
  - …if and only if…
  - It is not the case that…

Parse trees

- A parse tree depicts the way in which complex sentences are gradually built up from atomic ones.

Parse tree



- The main connective of a complex sentence is the last connective used in constructing it. (Here highlighted in orange.)

Types of complex sentences

- We give special names to certain types of sentences based on their main connective.
- A conjunction has 'and' as a main connective. The inputs are known as conjuncts.
- A disjunction has 'or' as a main connective. The inputs are known as disjuncts.
- A conditional has 'if…then' as a main connective. The left-hand input is known as the antecedent. The right-hand input is known as the consequent.
- A negation has 'it is not the case that' as a main connective. The input is known as the negand.

Types of complex sentences (ctd.)

- A biconditional has 'if and only if' as a main connective. The inputs are respectively known as the left-hand term and right-hand term of the biconditional.

Rewriting complex sentences

- These names are *also* given to sentences that can be *equivalently rewritten* as sentences with the relevant main connective.

Conjunctions

Sylvia and Sophie like cheesecake.
$\Rightarrow$ Sylvia likes cheesecake and Sophie likes cheesecake.

Sylvia and Sophie got married.
$\not\Rightarrow$ Sylvia got married and Sophie got married.

One more mistake and he'll be fired.
$\not\Rightarrow$ He'll make one more mistake and he'll be fired.

Rewriting complex sentences (ctd.)

Conditionals

If Hewan will be there, Faisal will be there.
Faisal will be there if Hewan will be there.
Hewan will be there only if Faisal will be there.
⇒ If Hewan will be there, then Faisal will be there.

One more mistake and he'll be fired.
⇒ If he makes one more mistake, then he'll be fired.

Rewriting complex sentences (ctd.)

Negations

Not all Belgian administrators are very cheerful.
⇒ It is not the case that all Belgian administrators are very cheerful.

All Belgian administrators are not very cheerful.
⇏ It is not the case that all Belgian administrators are very cheerful.

A coincidence?

- Consider:

Cluedo

(1) Either Professor Plum did it, or Colonel Mustard did.
(2) It isn't the case that Professor Plum did it.

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(3) Colonel Mustard did it.

Keys

(1) Either my car keys are at home, or they are in the office.
(2) It isn't the case that my car keys are at home.

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(3) My car keys are in the office.

A coincidence? (ctd.)

Still out and about

(1) If she was back, then the lights would be on.
(2) It is not the case that the lights are on.

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(3) It isn't the case that she is back.

Departure

(1) If he had been planning on leaving, then he would have contacted Sam.
(2) It isn't the case that he contacted Sam.

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(3) It isn't the case that he was planning on leaving.

A coincidence? (ctd.)

Hiding

(1) He either stayed in Damascus or went to Aleppo.
(2) If he stayed in Damascus, he will have been arrested.
(3) If he went to Aleppo, he will have been murdered.

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(4) He will have been arrested or murdered.

Bar Rumba

(1) Either it'll be crowded or it'll be empty.
(2) If it's crowded, it'll be too hot.
(3) If it's empty, it'll be boring.

---

(4) It'll either be boring or too hot.

Sentential form

- All these arguments are valid.
- But their validity doesn't seem to depend on *what they are about*.
- It looks like it's just the form (pattern, structure,...) that they instantiate that matters.

  (Note: whether or not the arguments are *sound*, however, will also depend on subject matter!)
- There are different *kinds* of argument forms, but the forms that account for the validity of our examples are all of the same kind.
- They are sentential forms: structural properties that do not depend on the internal details of the atomic sentences.

Building blocks

- The sentential forms of an argument are represented by means of a special language: a sentential formal language.
- We shall call it $\mathcal{L}_S$.
- Its sentences are built from three kinds of components.
  - a set of lower-case letters, called 'atomic sentences': $p, q, r, \ldots$
  - a set of connective symbols: $\&, \vee, \supset, \equiv, \sim$
  - a pair of bracket symbols '(', and ')'.

Building blocks (ctd.)

- The connective symbols correspond to our main English connectives, as follows:

English	$\mathcal{L}_S$	common variants
...and...	$\&$	$\wedge$
...or...	$\vee$	
if..., then...	$\supset$	$\rightarrow$
...if and only if...	$\equiv$	$\leftrightarrow$
it is not the case that...	$\sim$	$\neg$

Syntax

- Like any language, our sentential formal language has a syntax or set of grammatical rules.
- A well-formed sentence (wfs) is a sentence whose construction obeys these rules.
- Rulebook:
  1. Any atomic formula is a wfs.
  2. If P is a wfs, $\sim P$ is a wfs.
  3. If P and Q are wfs's, so are $(P\&Q)$, $(P \vee Q)$, $(P \supset Q)$ and $(P \equiv Q)$.
  4. Nothing else is a wfs.
- Note: it is common to omit the outmost brackets of a wfs, so that we write, for e.g., '$(p\&q)\vee \sim r$' rather than '$((p\&q)\vee \sim r)$'.

Wfs: some examples

A wfs

$((p\&q) \supset r)$ is a wff:

Since p, q and r are atomic formulae, they are wfs's (by 1)
Since p and q are wfs's so too is $(p\&q)$ (by 3)
Since $(p\&q)$ and r are wfs's, so too is $((p\&q) \supset r)$ (by 3)

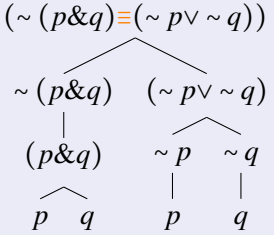
Not wfs's

$(p\&q \supset r)$, pq and $(\supset r)$ are *not* wfs's.

Parse trees again

- Yet again, parse trees can provide a nice representation of the stages involved in constructing a wfs from a set of atomic sentences.

Parse tree



From English to $\mathcal{L}_S$

- We have a language with which to represent sentential forms.
- So how do we 'translate' from English to $\mathcal{L}_S$, i.e. hook up an argument in English with its corresponding forms?
- Procedure:
  (1) Replace each atomic subsentence of the argument by a lowercase letter, such that any two *different English atomic sentences* are replaced by two *different letters*.
  (2) For each sentence in the argument, gradually replace each English connective by its symbolic counterpart, working your way up from the bottom of the parse tree and paying attention to adding the appropriate brackets.
- If a form can be obtained from an argument via this procedure, then the argument instantiates that form.

Most descriptive sentential forms

- It turns out that our procedure allows arguments to have *more than one* sentential form.
- This is because step (1) doesn't require that any two *same English atomic sentences* be replaced by the *same letter*.
- And this means that we sometimes have more than one option when replacing the English atomic sentences.
- An example...

Most descriptive sentential forms (ctd.)

Cluedo

(1) Either Professor Plum did it, or Colonel Mustard did.
(2) It isn't the case that Professor Plum did it.

---

(3) Colonel Mustard did it.

Form (a):

(1) $p \vee q$
(2) $\sim r$

---

(3) s

Form (b):

(1) $p \vee q$
(2) $\sim p$

---

(3) q

Most descriptive sentential forms (ctd.)

- Adding the additional requirement that any two same English atomic sentences be replaced by the same letter yields the most descriptive sentential form.
- With the additional requirement, English atomic sentences and letters are placed in one-to-one correspondence.
- An argument can only have *one* most descriptive sentential form.
- In *Cluedo*, the most descriptive sentential form was form (b).

Next session

- Topic: validity of sentential forms and truth tables.