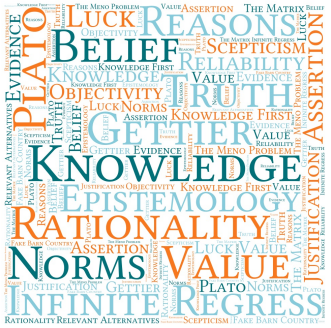


Knowledge, Reason & Belief

Jake Chandler

11. The Wisdom of the Crowds

LA TROBE UNIVERSITY

Last week

- The possible relevance to philosophy of the practise of polling people's intuitions on scenarios (XPhi)
- Two informal principles supporting this: Principle of Agreement and Principle of Reliability
- Justification for the Principle of Agreement in the form of the Condorcet Jury Theorem and associated results
- Presentation and critical discussion of work that
  (1) Reports polls allegedly demonstrating an effect of 'irrelevant factors' on intuitions
  (2) Concludes from the data that our intuitions (even as philosophers) wrt the relevant scenarios ought not be trusted

Introduction

This week

- More on the issue of how to aggregate (or not!) the opinions of a group of people
- This is an epistemological question with an obviously *far* broader relevance than the mere issue of XPhi
- Think:
  - Voters
  - Jurors
  - Scientific experts
  - Sensors
  - Databases

Some terminology

- An agenda is a list of propositions (sentences), such for each proposition on the list, each individual will cast a true/false vote

  Example: $\langle P, Q, (P \& Q) \leftrightarrow R, R \rangle$
- A judgment is a list of true (T) / false (F) / undecided (–) verdicts on each proposition of the agenda

  Example: $\langle T, T, F, - \rangle$
- A judgment aggregation procedure is a method that takes a list of individual judgments of the n members of a group and returns, as an output, a group-level judgment

  Example: majority, super-majority, unanimity, dictatorship
- We'll assume individual judgments to be logically consistent and complete (i.e. returning either T or F on all propositions)

Problems for majoritarians (and others)

The 'Discursive Dilemma'

- Last time:

  Under certain conditions, majority voting on agendas with a single proposition yields very reliable results (Condorcet Jury Theorem)
- Now the bad news:

  Majority voting on agendas that contain a set of logically connected propositions can lead to inconsistent group judgments
- Original observation: Kornhauser & Sager (1986) ('Doctrinal Paradox') in legal studies
- Introduced to philosophy by Pettit (2001) ('Discursive Dilemma')
- Related to a famous observation of Condorcet's regarding majority voting on preferences

The 'Discursive Dilemma' (ctd)

- Consider a panel giving advice on various claims relating to global warming, with E = Emissions are above some specific threshold and T = Global temperature will increase by some specific number of degrees by a specific date

	E	$E \rightarrow T$	T
Person 1	T	T	T
Person 2	T	F	F
Person 3	F	T	F
Majority	T	T	F

A general result

- As List points out, there are *lots* of conceivable aggregation procedures:

  For an agenda with only one proposition, there are 2^n combinations of individual judgments, each of which can be mapped to a different output: 2^{2^n} procedures in total!

- Question: Might we able to find one that fares better?
- A prima facie worrying result:

  Any procedure that satisfies a number of seemingly compelling conditions is *also* liable to generate inconsistent outputs

A general result (ctd)

- Here then is an early theorem from List and Pettit (2002)

  If an aggregation procedure satisfies Decisiveness, Anonymity and Systematicity, then it doesn't satisfy Consistency

- Note that related 'impossibility' results have long been established wrt preference aggregation (e.g. Arrow's Theorem)
- At least one thing has to go; I'll consider, in the following order:
  - Anonymity
  - Consistency
  - Systematicity
  - Decisiveness

A general result (ctd)

- Consider:

  Consistency: The aggregation procedure always generates as output *consistent* judgments on the propositions on the agenda.

  Decisiveness: The aggregation procedure always generates as output *complete* judgments on the propositions on the agenda.

  Anonymity: All individuals' attitudes are given equal weight in determining the group attitudes.

  Systematicity: The collective judgment on each proposition on the agenda depends only on the individual judgments on it, and the pattern of dependence is the same across propositions.

Escape routes

Relaxing Anonymity

- Pauly and van Hees (2006) show that that the result *still* holds when then condition is weakened to:

  Non-Dictatorship: It is not the case that there exists a member of the group whose judgment invariably determines the group judgment (dictator or inverse dictator)

- An inverse dictatorship is obviously bad if we assume that all voters are minimally reliable
- A dictatorship is fine, but the end result will only be as reliable as the dictator: aren't we missing out on some possible crowd wisdom?

Relaxing Consistency

- If coherence can't be secured given *all* inputs, it could remain the case that things pan out fine on a big subset of these
- List notes that majority voting yields consistent output in for a class of inputs that intuitively seem 'cohesive'
- One central set of cases:

  Unidimensional alignment: 'the group members can be aligned from left to right…such that for ever proposition on the agenda, the individuals accepting the proposition are either all to the left, or all to the right, of those rejecting it'

Relaxing Consistency (ctd)

- Visual illustration:

	1	2	3	4	5
E	F	F	F	F	T
$E \rightarrow T$	T	T	T	T	F
T	T	T	F	F	F

- Explanation, for when n is odd:

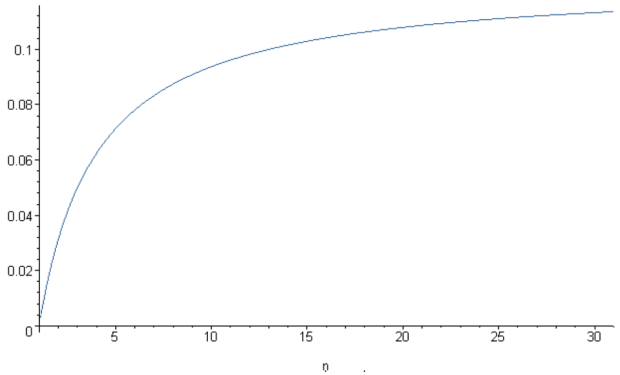
  The group judgment will correspond to the (consistent) judgment of the median voter (above: #3)

Relaxing Consistency (ctd)

- Question: right, but what *proportion* of all inputs do these kinds of 'cohesive' inputs correspond to?
- List (2005) calculates the probability of inconsistency given n in a case analogous to the one we started with, under the assumption that all possible relative frequencies of sets of judgments are equiprobable ('impartial anonymous culture')

Relaxing Consistency (ctd)

- The probability looks pretty high to me:



Relaxing Systematicity

- One popular way of doing this involves majority voting on a restricted subset of the agenda
- Sometimes, it makes sense to divide the agenda into
  - a set of premises (e.g. $\{E, E \rightarrow T\}$), that entail
  - a set of conclusions (e.g. $\{T\}$)

  so that, ultimately, we only really care about the conclusions
- Another way of putting it:

  > Sometime we have a complex problem (e.g. whether or not T) that is divisible into simpler problems (e.g. whether or not E and whether or not $E \rightarrow T$)

Relaxing Systematicity (ctd)

- We then have the options of majority voting on either
  - just the premises (then taking the stance on the conclusions that is entailed by the majority view on the premises)
  - just the conclusions (suspending judgment on the premises)

	E	$E \rightarrow T$	T
Person 1	T	T	T
Person 2	T	F	F
Person 3	F	T	F
Premise-based	T	T	T
Conclusion-based	–	–	F

Relaxing Systematicity (ctd)

- Clearly, both methods satisfy Anonymity
- Given Anonymity, Systematicity amounts to

  > The group judgment on all proposition is determined in the same way by the frequencies of votes on that proposition (e.g. true if freq. =0.75 and false if not)

- It is then easy to see that both methods violate the requirement
  - *Conclusion-based*: T would still have been judged false and E left undetermined, if E had, like T, received 1/3 of the vote
  - *Premise-based*: 2/3 of the voters judged E to be true, and 2/3 judged T to be false, but the group judged both E and T to be true

Relaxing Systematicity (ctd)

- Note that neither procedure satisfies Decisiveness, since majority voting in general doesn't (if n is even, we can get a tie; more on this later)
- Consistency is also satisfied in both cases:
  - *Premise-based*: if the premises are logically independent
  - *Conclusion-based*: if the conclusions are logically independent
- Ok, but how likely are these procedures to yield true judgments on the entire agenda? See Hartmann & Sprenger (2012)

Relaxing Systematicity (ctd)

- What if we *can't* divvy things up into premises and conclusions?
- One option:

  > Majority vote on the atomic propositions, i.e. logically simple propositions associated with the agenda (the letters inside the formulae)

- The truth values of these atomic sentences then logically determine the truth value of all sentences on the agenda

  Example: $E \rightarrow T$ is true iff either E is false or T is true
- Note: this only works if the atomic propositions *are on the agenda*; if they aren't, a more general approach, the distance-based approach mentioned by List, can be used

Relaxing Systematicity (ctd)

- Here is the verdict:

	E	$E \rightarrow T$	T
Person 1	T	T	T
Person 2	T	F	F
Person 3	F	T	F
Premise-based	T	T	T
Conclusion-based	–	–	F
Atom-based	T	F	F

Relaxing Systematicity (ctd)

- Majority voting on atoms obviously satisfies Anonymity
- It's then easy to see that it violates Systematicity:

  > 2/3 of the voters judged $E \rightarrow T$ to be true, and 2/3 judged T to be false, but the group judged both E and T to be true

- It also guarantees Consistency, since atomic propositions are logically independent
- Decisiveness fails again, since majority voting can lead to ties if n is even
- How likely is this method to yield true judgments on the entire agenda? See again Hartmann & Sprenger (2012) for a discussion and comparison with premise- and conclusion-based procedures

Relaxing Systematicity (ctd)

- One noteworthy thing about this procedure is that it violates

  Unanimity: The group judgment should respect unanimity

	P	Q	R	$P \vee Q \vee R$
Person 1	T	F	F	T
Person 2	F	T	F	T
Person 3	F	F	T	T
Atom-based	F	F	F	F

- Is that bad?

Relaxing Decisiveness

- The above procedures relax Decisiveness *and* Systematicity
- What about relaxing *only* Decisiveness? (E.g. majority, supermajority)
- List & Pettit's theorem doesn't predict trouble here…but the Discursive Dilemma highlights problems with these!!
- Could there be a general result that explains what the problem is?
- We've seen that Anonymity and Systematicity give us the requirement that group judgment wrt to a proposition is determined by the frequencies of votes on that judgment
- But this doesn't tell us *anything* about *how* it is determined

Relaxing Decisiveness

- One last condition, then:

  Monotonicity: If the group judges a proposition to be true, it would have still judged that proposition to be true had one of the voters who voted it to be false voted it to be true instead

- Given Anonymity and Systematicity, this amounts to

  If (i) a proposition P would be judged true by the group on the basis of a frequency of votes F_1 and (ii) P^* were to receive a frequency of votes $F_2 \geq F_1$, then (iii) P^* would be judged true by the group

- This should look quite familiar…
- I'll leave it as an exercise to guess what the result might be

Next week

- Final course wrap-up (no seminar on Wednesday!)
- Bring questions and comments
- Also happy to discuss presentations

References

- Kornhauser, L.A., & L.G. Sager 1986: Unpacking the Court. *Yale Law Journal* 96: 82–117.
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- Pauly, M. & M. van Hees 2006: Logical Constraints on Judgement Aggregation. *Journal of Philosophical Logic* 35(6):569–585
- Pettit, P. 2001: Deliberative Democracy and the Discursive Dilemma. *Philosophical Issues* 11: 268–99.