Prospects for a General Theory of Belief Change

James Delgrande
Simon Fraser University
Canada
jim@cs.sfu.ca

Introduction

Belief Change:

- The area of belief change studies how an agent may rationally change its beliefs in the face of new information.
- The *AGM approach* is the central theory of belief change.
  - We'll be working within the AGM framework
- Two primary belief change functions:
  - *revision* (where an agent accommodates new information)
  - *contraction* (where an agent's ignorance increases)

Belief Change: Revision

In belief revision, an agent

- incorporates a new belief ϕ, while
- maintaining consistency (unless $\vdash \neg\phi$).

Thus an agent may have to remove beliefs to remain consistent.

Problem: Logical considerations alone are not sufficient to determine a revision function.

☞ Similar considerations apply to contraction.

Belief Change: Characterizations

- In the AGM approach, change functions are studied at the *knowledge level*.
- An agent's beliefs are given by a *belief set* or deductively closed set of formulas.
- Notation:
  - K: Belief set
  - $K * \phi$: Revision of K by ϕ
  - $K \dot{-} \phi$: Contraction of K by ϕ
  - Also:
    $K + \phi = Cn(K \cup \{\phi\})$, the *expansion* of K by ϕ.

Belief Change: Characterizations

Belief change functions are captured by two means:

I Constructions:

Abstract specification of how to construct a belief change function.

- E.g. faithful ranking/system of spheres, remainder sets, epistemic entrenchment ordering

II Postulates:

Criteria that constrain any "rational" function.

- E.g. $\phi \in K * \phi$ or If $\nvdash \phi$ then $\phi \notin K \dot{-} \phi$.

Representation Result:

- Show that a construction $\approx$ a postulate set.

Belief Change

Revision and contraction may be taken as interdefinable via:

- The Levi identity: $K * \phi = (K \dot{-} \neg\phi) + \phi$
- The Harper identity: $K \dot{-} \phi = K \cap (K * \neg\phi)$

Belief Change

The various characterisations of revision and contraction are, broadly speaking, shown to be equivalent.

This includes equivalences given by

- a representation result, or
- the Levi/Harper identities, or
- the inter-specification of constructions like faithful rankings, remainders, epistemic entrenchment

Conclude: AGM change is a well-studied, unified, area of KR.

Belief Change

However:

- The AGM approach assumes the underlying logic contains classical propositional logic (PC).
- But *many* AI/KR approaches don't subsume PC.
  - E.g., Horn clause reasoners, (extended) logic programs, description logics, etc.
- This has led to *lots* of logic-specific approaches to belief change
  - E.g. in specific DLs, ASP, Horn programs, etc.
  - Most often the full set of (revision or contraction) postulates isn't obtained.

☞ It is of interest to investigate belief change with respect to *arbitrary* logics.

Belief Change in Arbitrary Logics

This talk:

- describes a recent approach to revision in any logic
  - ☞ joint work with Pavlos Peppas and Stefan Woltran
- describes preliminary work on contraction in any logic
  - ☞ builds on work with Renata Wasserman
- outlines what I feel this says about belief change in general.

Overview

- Introduction (just completed)
- Arbitrary logics
- Revision in arbitrary logics
- Contraction in arbitrary logics
- Conclusion

Arbitrary Logics

For an arbitrary logic, we have three primitive entitites:

- A nonempty *language* $\mathcal{L}$.
- A finite, nonempty set $\mathcal{M}$ of *possible worlds* or simply *worlds*.
- A function f from $\mathcal{L}$ to $2^{\mathcal{M}}$.

  For $\phi \in \mathcal{L}$, $w \in \mathcal{M}$, write $w \models \phi$ for $w \in f(\phi)$.
- And that's it!

☞ While very basic, this defines a *Tarskian consequence relation*.

Recall: Belief Revision

Standard construction: faithful ranking.

Faithful ranking: Assign to each belief set K

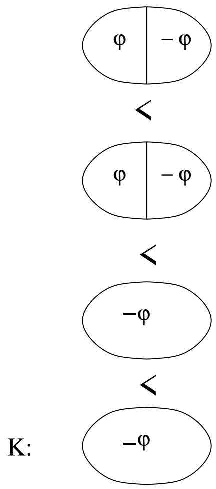
- a *total preorder* $\preceq_K$ over possible worlds ...
- ...such that models of K are minimal in the preorder.

☞ The preorder gives the relative *plausibility* of a world.

Define: $Mod(K * \phi) \ = \ \min(Mod(\phi), \preceq_K)$.

☞ $K * \phi$ is characterized by the most plausible ϕ worlds according to the agent.

Faithful Ranking



AGM Revision Postulates

(K*1) $K * \phi = \mathcal{C}n(K * \phi)$
(K*2) $\phi \in K * \phi$
(K*3) $K * \phi \subseteq K + \phi$
(K*4) If $\neg\phi \notin K$ then $K + \phi \subseteq K * \phi$
(K*5) $K * \phi$ is inconsistent only if ϕ is inconsistent
(K*6) If $\phi \equiv \psi$ then $K * \phi = K * \psi$
(K*7) $K * (\phi \wedge \psi) \subseteq K * \phi + \psi$
(K*8) If $\neg\psi \notin K * \phi$ then $K * \phi + \psi \subseteq K * (\phi \wedge \psi)$

☞ These postulates exactly capture revision defined in terms of faithful rankings.

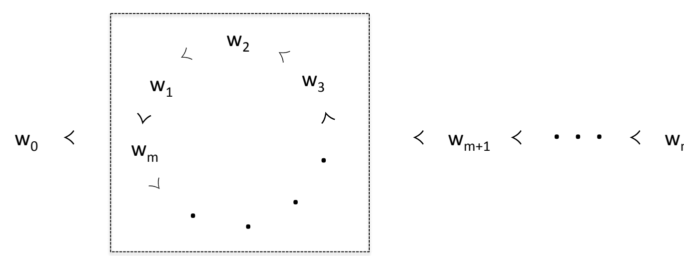
Revision in Arbitrary Logics

☞ Let's see what happens if we try to apply the AGM approach in an arbitrary logic.

- We can adopt the AGM approach wholesale, except
  - replace the second argument by a set of formulas: $K * A$ (since we may not have conjunction)
  - rewrite the postulates so that conjunction and negation aren't mentioned, and
  - don't assume that the underlying logic contains PC.
- Not unexpectedly, we run into problems.

General Revision: Problem

Consider the following pseudo-order, where w_0 represents the agent's beliefs and $\prec$ is reflexive only:



- The function $*$ induced from $\prec$ satisfies (K*1) - (K*8).

Problem: Without PC, the postulates are too weak to rule out "circular" $\prec$-worlds.

General Revision: Another Problem

Some postulates may not be satisfied in a faithful ranking.

- Specifically, a faithful ranking may violate (K*7) and (K*8).

Problem: There are sets of worlds for which there is no corresponding theory.

- E.g., in the logic of Horn clauses, there is no Horn theory H such that $Mod(H) = \{a\bar{b}, \bar{a}b\}$.

General Revision: Solution

Without PC:

- the AGM postulates can't capture faithful rankings; and
- faithful rankings can't capture the AGM postulates.

Solution: Strengthen both parts.

Specifically:

- add a condition to restrict faithful rankings; and
- add a postulate to the set of AGM postulates.

Key point:

- In propositional logic, these additions are *redundant*.
- Hence, this solution is a *generalization* of AGM revision.

General Revision: Ranking Functions

We restrict faithful rankings to *regular* rankings.

Idea:
In a ranking, require for any set of formulas A, that

$$\min(Mod(A), \preceq)$$

is representable by a set of formulas.

Definition

- A set of worlds W is *elementary* iff there is a set of formulas A such that $W = Mod(A)$.
- A preorder $\preceq$ is *regular* iff for every set of formulas A

  $\min(Mod(A), \preceq)$ is elementary.

General Revision: New Postulate

We add a schema that informally states that

$$\text{if} \quad w_0 \preceq w_1 \preceq \cdots \preceq w_n \preceq w_0 \quad \text{then} \quad w_0 \preceq w_n.$$

☞ Roughly: $\prec$-cycles are ruled out in a ranking.

Formally this is expressed:

(Acyc) If for $0 \le i < n$ we have $(K * A_{i+1}) + A_i \nvdash \bot$, and $(K * A_0) + A_n \nvdash \bot$, then $(K * A_n) + A_0 \nvdash \bot$.

We have:

- (Acyc) is a *logical consequence* of the AGM postulates in PC.
- (Acyc) is *independent* of the AGM postulates in the general framework.

Representation Result

These changes are sufficient for capturing general revision:

Theorem:
A revision operator $*$ satisfies (K*1) – (K*8) and (Acyc)

iff

there is a faithful ranking that maps each belief set K to a regular total preorder $\preceq$ such that

$$Mod(K * A) = \min(Mod(A), \preceq)$$

Examples

Some instances of the approach have been developed, including

- Horn clause revision
- Answer set programs
- Literal revision

Conclude: AGM-style revision extends to arbitrary logics

Ask: What about contraction?

Prospects for AGM-Style Contraction in Arbitrary Logics

Obvious suggestion:
Define contraction in terms of revision via the Harper Identity:

$$K \dot{-} \phi = K \cap (K * \neg\phi)$$

Problem:
$\neg\phi$ may not be in the language (e.g. Horn clauses)

- A similar problem would arise with the Levi Identity for defining revision in terms of contraction.

Recall: Contraction

A standard construction for contraction functions uses *remainders*.

For belief set K and formula ϕ:

- A *remainder* is a maximal subset of K that doesn't imply ϕ.
- *Maxichoice contraction*: Let $K \dot{-} \phi$ be some remainder
- *Partial meet contraction*: Let $K \dot{-} \phi$ be the intersection of some set of remainders
- *Full contraction*: Let $K \dot{-} \phi$ be as in partial meet, but choose the set of remainders via a transitive relation on 2^K

☞ Unfortunately, we run into problems in arbitrary logics.

- We'll look at Horn clause logic as an example.

Problem: Contraction in Horn Theories

- Remainder sets are too coarse for contraction in Horn theories
  - Consider $\mathcal{P} = \{a, b, c\}$, $K = \mathcal{C}n(\{a, b\})$, $\phi = \{a, b\}$,
  - We cannot obtain $\mathcal{C}n(\{a, c \to b\})$ as a possible contraction.
- Instead, use *weak remainders*.
  - Informally, add a countermodel of ϕ to the models of K
  - More formally,
    - a *world set* is the set of formulas true at some model. (So like a maximum consistent set.)
    - $|\psi|$ is the set of world sets comtaining ψ.
  - Then a weak remainder is some K' where

$$K' = K \cap C \quad \text{for} \quad C \in |\emptyset| \setminus |\phi|$$

☞ This yields a satisfactory account of contraction in Horn logic.

Contraction in Arbitrary Logics

Intuition:

Extend this work to arbitrary logics.

- So far it seems to work.
- As before, define contraction in terms of a set of formulas A

Weak Remainder

A definition of a (weak) remainder set is given by:

Definition

$K \Downarrow A$ is given by:

If $A \not\subseteq K$ *then* $\quad K \Downarrow A = \{K\}$

Otherwise

$$K' \in K \Downarrow A \quad \textit{iff} \quad K' = K \cap C \quad \textit{for some } C \in |\emptyset| \setminus |A|.$$

Partial Meet Contraction: Postulates

We obtain the following postulates characterizing partial meet contraction:

(K-1) $K \dot{-} A$ is a belief set.

(K-2) If not $\vdash A$ then $A \notin K \dot{-} A$.

(K-3) $K \dot{-} A \subseteq K$.

(K-4) If $A \notin K$ then $K \dot{-} A = K$.

(K-5') If $\vdash A$ then $K \dot{-} A = K$

(K-5") If $\phi \in K \setminus K \dot{-} A$ then $\exists C$ s.t. $C \in |\emptyset| \setminus |A|$ such that $\phi \notin C$ and $K \dot{-} A \subseteq C$

(K-6) If $A \equiv B$ then $K \dot{-} A = K \dot{-} B$.

Partial Meet Contraction

- The postulate

  (K-5') If $\vdash A$ then $K \dot{-} A = K$

  is redundant in the presence of PC (implied by recovery).
- The postulate

  (K-5") If $\phi \in K \setminus K \dot{-} A$ then $\exists C$ s.t. $C \in |\emptyset| \setminus |A|$ such that $\phi \notin C$ and $K \dot{-} A \subseteq C$

  expresses the maximality of $K \dot{-} A$:

  If ϕ is dropped from K in contracting by A, then this is because some world set C taking part in the contraction does not satisfy ϕ.
- With PC, (K-5") yields the recovery postulate.

General Contraction

- AGM postulates (K-7) and (K-8), and (Acyc) are sound wrt a a selection function based on a preference order over 2^K.
- *Conjecture*: Complete wrt the previous postulates
  + (K-7), (K-8), (Acyc).
- If the conjecture holds, obtain a reformulation of AGM revision and contraction in any logic.

Discussion

We have addressed belief change in *any* logic.

- Needed to deal with belief change in approaches that don't subsume PC.
- These results *extend* (rather than *modify*) the AGM approach.
- These results are applicable to any logic.

☞ However, from this perspective, the AGM landscape changes significantly

Discussion

☞ Without PC, the uniformity of the AGM approach is no longer present.

While we retain representation results for revision and contraction,

- revision and contraction appear to be distinct operations
  - constructions are based on differing intuitions.
  - appear to not be interdefinable
  - unclear how (or *if*) they relate in the general case.
- relations between constructions (e.g. faithful rankings vs. remainders) appear to break down.

Discussion

☞ These results suggest that belief change is best regarded as a *semantic* rather than proof-theoretic notion.

- The original AGM approach emphasised contraction and deductive inference.
    - E.g. emphasis on belief sets, and on syntactic constructions like remainders and epistemic entrenchment
    - These days, revision seems to be the dominant operator, and is based on semantic constructions.
- In arbitrary logics, syntactic constructs seem to be too weak.
    - E.g. a proof-theoretic account of remainders is too coarse in logics weaker than PC.
    - Suggests that remainders are best reconstituted from a model-theoretic viewpoint
    - Similarly, have problems with EE