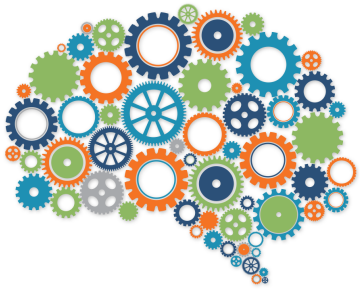

Critical Thinking & Innovation

Jake Chandler

Deductive Reasoning, Part 2

LA TROBE UNIVERSITY

Last week

- The notion of deductive validity (and soundness)
- The idea that arguments can be valid just because of the form of the sentences
- Two types of form: sentential and subsentential
- Sentential form: the way in which complex sentences are built up from atomic ones using connectives

Representing sentential form

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Representing sentential form

- The sentential form of an argument is represented using special notation (the "language of sentential logic")
- This notation is widely used in philosophy (also comp. sci.)
- Its sentences are built from:
  - upper-case letters, each representing a given atomic sentence $P, Q, R, \ldots$
  - symbols, representing connectives: $\wedge, \vee, \rightarrow, \leftrightarrow, \neg$
  - brackets: ()

Symbol translations

English	Sentential logic
…and…	$\wedge$
…or…	$\vee$
if…, then…	$\rightarrow$
…if and only if…	$\leftrightarrow$
it is not the case that…	$\neg$

From English to Sentential Logic

- How do we 'translate' from English to sentential logic?
  1. If required, rewrite the sentences in a format that "brings out" the sentential form
  2. Set up a dictionary, establishing a one-to-one correspondence between the English atomic sentences and various letters
  3. Replace each atomic sentence of the argument by its corresponding letter
  4. For each sentence, gradually replace each English connective by its symbolic counterpart, working up from the bottom of the tree (adding brackets around each new representation of a complex subsentence)

Single sentence example

Example

If you go left, then if I go right, then we'll collide.

1. No need to rewrite here
2. Make dictionary:
   P: You go left; Q: I go right; R: We'll collide
3. Replace atomic sentences:
   If P, then if Q, then R.
4. Replace connectives:
   If P, then $(Q \rightarrow R)$
   $P \rightarrow (Q \rightarrow R)$

Argument example 1: *Modus Ponens*

Example: Intergalactic Renegades

(P_1) If Jake teaches CTO, then he's an Intergalactic Renegade
(P_2) Jake teaches CTO

(C) Jake is an Intergalactic Renegade

Sentential form: Modus Ponens
(P_1) $P \rightarrow Q$
(P_2) P

(C) Q

Argument example 2: *Modus Tollens*

Example: Still out and about

(P_1) If he was back, then the lights would be on.

(P_2) It is not the case that the lights are on.

(C) It isn't the case that he is back.

Sentential form: Modus Tollens

(P_1) $P \to Q$

(P_2) $\neg Q$

(C) $\neg P$

Argument example 3: *Constructive Dilemma*

Example: Nightclub

(P_1) Either it'll be crowded or it'll be empty.

(P_2) If it's crowded, then it'll be too hot.

(P_3) If it's empty, it'll be boring.

(C) It'll either be boring or too hot.

Sentential form: Constructive Dilemma

(P_1) $P \vee Q$

(P_2) $P \to R$

(P_3) $Q \to S$

(C) $R \vee S$

Argument example 4: *Disjunctive Syllogism*

Example: Cluedo

(P_1) Either Professor Plum did it, or Colonel Mustard did.

(P_2) It isn't the case that Professor Plum did it.

(C) Colonel Mustard did it.

Sentential form: Disjunctive Syllogism

(P_1) $P \vee Q$

(P_2) $\neg P$

(C) Q

Argument example 5: *Hypothetical Syllogism*

Example: Up Late

(P_1) If you stay up late, then you'll be tired tomorrow.

(P_2) If you are tired tomorrow, then you'll be late for class.

(C) If you stay up late, then you'll be late for class.

Sentential form: Hypothetical Syllogism

(P_1) $P \to Q$

(P_2) $Q \to R$

(C) $P \to R$

Tautologies and contradictions

- Two important related concepts:

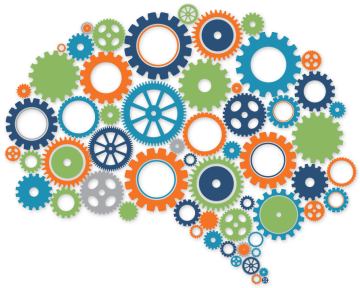
  Tautology: A sentence that had to be true (couldn't possibly have been false)

  Contradiction: A sentence that had to be false (couldn't possibly have been true)

- These concepts can be used to define validity
- An argument from $P_1 \wedge P_2 \wedge \ldots P_n$ to C is valid if and only if:

  $(P_1 \wedge P_2 \wedge \ldots P_n) \rightarrow C$ is a tautology

  $P_1 \wedge P_2 \wedge \ldots \wedge P_n \wedge \neg C$ is a contradiction



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Sub-sentential form

Further validities

- The validity of *many* arguments isn't due to sentential form

Example: Hippos

(P_1) All hippos are bad-tempered.

(P_2) Andrew is a hippo.

(C) Andrew is bad-tempered.

Sentential form:

(P_1) P

(P_2) Q

(C) R

Further validities (ctd.)

Example: Cats

(P_1) Some cats are not friendly.

(P_2) All cats are mammals.

(C) Some mammals are not friendly.

Sentential form:

(P_1) P

(P_2) Q

(C) R

Sub-sentential form

- But in the cases we've just seen, this validity is *still* due to form

Example: Hippos

(P_1) All hippos are bad-tempered.

(P_2) Andrew is a hippo.

(C) Andrew is bad-tempered.

Example: Corners

(P_1) All square have four corners.

(P_2) This shape is a square.

(C) This shape has four corners.

Subjects, predicates, quantifiers

- The validity is due to sub-sentential form: we need to look 'inside' the atomic sentences
- The sub-sentential form of an argument is also represented using special notation (the "language of predicate logic")
- Full details are beyond the scope of CTO but lets handle our 2 examples…

Subjects and predicates

- The second premise of the 'Hippo' argument:

  'Andrew is a hippo.'

  involves:
  - a name ('Andrew'), referring to a particular thing
  - a predicate ('…is a hippo'), to saying something about what that thing is like
- We represent subject-predicate structure using
  - lower-case letters ($a, b, c, \ldots$) for names
  - upper-case letters ($F, G, H, \ldots$) for predicates
- Our premise could be: $H(a)$

Quantifiers

- The other premises of the 'Hippo' and 'Cat' arguments:

  'All hippos are bad-tempered.'

  'Some cats are not friendly.'

  'All cats are mammals.'

  aren't about a *particular* individual: no proper names here…
- They are statements about *quantities* of individuals that use quantifiers
  - 'all'
  - 'some'
  - 'most'
  - 'few'

Universal quantifiers

- What exactly do we *mean* by

  'All hippos are bad-tempered.'
- Plausible gloss:

  It is true of *any* thing, call it 'x', that if x is a hippo, then x is bad tempered.
- The representation of the premise in predicate logic has that structure:

  $(\forall x)(H(x) \rightarrow B(x))$
- Note: x is known as a "variable" and $\forall$ as a "quantifier symbol" (the "'universal quantifier")

Existential quantifiers

- Similarly, consider:

  'Some cats are not friendly.'
- Gloss:

  It is true of *at least one* thing, call it 'x', that it is a cat and that it is not friendly.
- Again, same structure in the predicate logic representation:

  $(\exists x)(C(x) \wedge \neg F(x))$
- Note: $\exists$ is also a quantifier symbol (the "'existential quantifier")

Example 1

Example: Hippos

(P_1) All hippos are bad-tempered.

(P_2) Andrew is a hippo.

(C) Andrew is bad-tempered.

Sub-sentential form:

(P_1) $(\forall x)(H(x) \rightarrow B(x))$

(P_2) $H(a)$

(C) $B(a)$

Example 2

Example: Cats

(P_1) Some cats are not friendly.

(P_2) All cats are mammals.

(C) Some mammals are not friendly.

Sub-sentential form:

(P_1) $(\exists x)(C(x) \wedge \neg F(x))$

(P_2) $(\forall x)(C(x) \rightarrow M(x))$

(C) $(\exists x)(M(x) \wedge \neg F(x))$