



Critical Thinking Innovation

JAKE CHANDLER

Deductive Reasoning, Part 3

INTRODUCING TRUTH TABLES

Spotting a valid sentential form

- So far:
 - a gold standard for premise-conclusion relations: validity
 - what formal validity is
 - two ways of describing form
- Sure, but how do I *recognise* formal validity?!
- Possibilities:
 - (1) see if I have a well-know valid or invalid form
 - (2) brainstorm a counterexample
- Here: easy method for *sentential* form (in logic: more efficient methods and methods for subsentential form)

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Truth-functionality

- Our connectives are commonly taken to have the following feature:
 - truth-functionality**: the truth value of the output is determined by (= is a function of) the truth value of the inputs.
- *Not all* connectives are truth-functional!! Examples:
 - 'It is well known that...'
 - 'It could have been the case that...'

Example

'Paris is the capital of France' (true)

'The heart of a shrimp is located in its head' (true)

'It's well known that Paris is the capital of France' (true)

'It's well known that the heart of a shrimp is located in its head'
(false!!!)

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Truth tables

- The way in which a certain connective determines the truth value of its output from the truth value of its inputs is summarised in a **truth table**.
- Here are the truth tables for our five connectives, using **0** for **false** and **1** for **true**
- Straightforward: conjunctions, negations, biconditionals
- Require comment: disjunctions, conditionals
- With all these tables in hand, we'll then be able to construct tables for *any* kind of sentence or indeed argument whatsoever.

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Conjunctions and negations

- A conjunction is true if and only if both its conjuncts are true. Otherwise it is false.

P	Q	$P \wedge Q$
1	1	1
1	0	0
0	1	0
0	0	0

- A negation is true if and only if its negand is false. Otherwise it is false.

P	$\neg P$
1	0
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Biconditionals

- A biconditional is true if and only if both its terms have the same truth value. Otherwise it is false.

P	Q	$P \leftrightarrow Q$
1	1	1
1	0	0
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Disjunctions

- A disjunction is true if and only if at least one of its disjuncts is true. Otherwise it is false.

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Conditionals

- A conditional is false if and only if its antecedent is true but its consequent is false. Otherwise it is true.

P	Q	$P \rightarrow Q$
1	1	1
1	0	0
0	1	1
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- Note: a conditional whose antecedent is false is considered true!
- This is a bit controversial
- Rationale: $P \rightarrow Q$ is argued to amount to $\neg P \vee Q$

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Step 1

Draw up a table: $r = 2^n$ rows, where n is the number of atomic sentences. (Here: $r = 4$.)

Column headers: atomic sentences on the left, complex sentence on the right.

Tables for longer sentences: Example 1

- The construction of a table for $((P \leftrightarrow Q) \wedge P) \rightarrow Q$:

P	Q	$((P \leftrightarrow Q) \wedge P) \rightarrow Q$
1	1	
1	0	
0	1	
0	0	

Step 2

Fill in the columns for the atomic sentences on the left, making sure to write down every possible combination of truth values.

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P	Q	$((P \leftrightarrow Q) \wedge P) \rightarrow Q$			
1	1	1		1	
1	0	1		0	
0	1	0		1	
0	0	0		0	

Step 3

Work your way up the parse tree, filling in the values for increasingly more complex sentences

- For atomic sentences: copy-paste the values under the letter
- For complex sentences: write the values under the main connective, using the relevant. tables

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0	0	0	1	0	0	0	0	1	0	

Some important terms

- Each row of the table represents a **valuation**: an assignment of truth values to the argument's sentences / subsentences that is consistent with the t. tables for the connectives.
- Think of these as **possible situations**
- Under the main connective of Example 1, we find a column of 1's: the sentence is true on all valuations.
- The sentence is a **tautology**.
- Under the main connective of Example 2, we find a column of 0's: the sentence is false on all valuations.
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Critical Thinking [^] Innovation

JAKE CHANDLER

Deductive Reasoning, Part 3

TRUTH TABLES FOR ARGUMENTS

Truth tables for arguments

- We can apply the method of truth tables to prove the validity or invalidity of a sentential argument form.
- We draw up, side by side, the sub-tables for the various sentences of the argument form.
- Table for the **disjunctive syllogism** (see Cluedo example):

P	Q	P	$\vee$	Q	$\neg$	P	Q
1	1	1	1	1	0	1	1
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Truth tables for arguments (ctd.)

- We then consider the question:
Is there a valuation that makes all the premises true but the conclusion false?
- If NO, then the form is valid.
- If YES, then the form is invalid.

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A valid form

- We can now see that the disjunctive syllogism is valid

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- The only rows that give us a false conclusion are the second and fourth ones. But in both cases, one of the premises is false
- Exercise: use tables to check the validity of our other valid sentential forms

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Some invalid forms: *Affirming a Disjunct*

- Recall our examples of common arguments with deductively invalid sentential forms (**formal fallacies**)

Example: Treats

(P₁) Isaac either got a cookie or got an ice-cream.

(P₂) Isaac got a cookie.

(C) Isaac didn't get an ice-cream.

Form: $P \vee Q, P$, therefore $\neg Q$ (aka **Affirming a Disjunct**)

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0	1	0	1	1	0	0	1
0	0	0	0	0	0	1	0

- Row 1 (P and Q both true): all the premises are true but the conclusion is false
- The valuation that this row represents is known as a **countermodel** (or counterexample)
- In Treats: Isaac's having both a cookie *and* an ice-cream

Some invalid forms: *Affirming a Disjunct* (ctd)

P	Q	P	$\vee$	Q	P	$\neg$	Q
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Some invalid forms: *Affirming the Consequent*

Example: Steroids

(P₁) If steroids were taken, the test will come out positive.

(P₂) Sam's test came out positive.

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Form: $P \rightarrow Q$, Q , therefore P (aka *Affirming the Consequent*)

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- Row 3 (P false, Q true): all the premises are true but the conclusion is false
- In Steroids: Sam's test came out positive, but he didn't take steroids (he's perhaps on some other medication)

Some invalid forms: *Affirming the Consequent* (ctd)

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Some invalid forms: *Denying the Antecedent*

Example: Penknife

(P₁) If Billy's dad found his penknife, he's in deep trouble.

(P₂) Billy's dad didn't find his penknife.

(C) Billy isn't in deep trouble.

Form: $P \rightarrow Q, \neg P$, therefore $\neg Q$ (aka *denying the antecedent*)

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Form: $P \rightarrow Q$, $\neg P$, therefore $\neg Q$ (aka denying the antecedent)

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P	Q	P	$\rightarrow$	Q	$\neg$	P	$\neg$	Q
1	1	1	1	1	0	1	0	1
1	0	1	0	0	0	1	1	0
0	1	0	1	1	1	0	0	1
0	0	0	1	0	1	0	1	0

- Row 3 again (P false, Q true): all the premises are true but the conclusion is false
- In Penknife: Billy's dad didn't find the penknife, but Billy is still in trouble (for some other reason)

Some invalid forms: *Denying the Antecedent* (ctd)

P	Q	P	$\rightarrow$	Q	$\neg$	P	$\neg$	Q
1	1	1	1	1	0	1	0	1
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What is said vs what is implied

- But *why* do we make these mistakes?!
- Same arguable root cause:

We are confusing what the premises *imply* with what the premises *say* and hence considering different arguments (which happen to be valid)

Example

You tell me that you think the candidate was funny and smart. I tell you “I definitely think he was funny”. I have told you one thing, but implied something rather stronger: that I think the candidate was funny *and that I think the candidate was not smart.*

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You ask me whether I've had some cookies yet. I tell you "Yes, actually, I have had some cookies.". I have said one thing but implied something stronger: that I have had some of the cookies *and that I have not had all of them.*

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Back to our fallacies: Asserting a Disjunct

- When we read $P \vee Q$, we might assume that it isn't the case that both P and Q are true
 $\Rightarrow$ Interpretation: $(P \vee Q) \wedge \neg(P \wedge Q)$
- On the alternative truth table, the argument is valid:

P	Q	$(P \vee Q) \wedge \neg(P \wedge Q)$						$P \rightarrow Q$		
1	1	1	1	1	0	0	1	1	1	1
1	0	1	1	0	1	1	1	0	0	1
0	1	0	1	1	1	1	0	0	1	0
0	0	0	0	0	0	1	0	0	0	1

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P	Q	$(P \vee Q)$	$\wedge$	$\neg$	$(P \wedge Q)$	P	$\neg$	Q
1	1	1	1	0	0	1	0	1
1	0	1	1	0	0	1	1	0
0	1	0	1	1	1	0	0	1
0	0	0	0	1	0	0	1	0

Back to our fallacies: Asserting a Disjunct

- When we read $P \vee Q$, we might assume that it isn't the case that both P and Q are true
 $\Rightarrow$ Interpretation: $(P \vee Q) \wedge \neg(P \wedge Q)$
- On the alternative truth table, the argument is valid:

P	Q	$(P \vee Q) \wedge \neg(P \wedge Q)$					P	$\neg Q$
1	1	1	1	0	0	1	1	0
1	0	1	0	1	1	1	0	1
0	1	0	1	1	1	0	1	0
0	0	0	0	0	1	0	0	1

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0	1	0	1	1	0	0	0	1
0	0	0	0	1	0	0	1	0

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1	0	1	1	0	1	1	1	0	0	1	1	0
0	1	0	1	1	1	1	0	0	1	0	0	1
0	0	0	0	0	0	1	0	0	0	0	1	0

Back to our fallacies: The remaining two fallacies

- When we read $P \rightarrow Q$, we might assume that there is no other sufficient condition for Q other than P
 $\Rightarrow$ Interpretation: $P \leftrightarrow Q$
- On the alternative truth table (here for Affirming the Consequent), the argument is valid:

P	Q	P	$\leftrightarrow$	Q	Q	P
1	1	1	1	1	1	1
1	0	1	0	0	0	1
0	1	0	0	1	1	0
0	0	0	1	0	0	0

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0	0	0	1	0	0	0