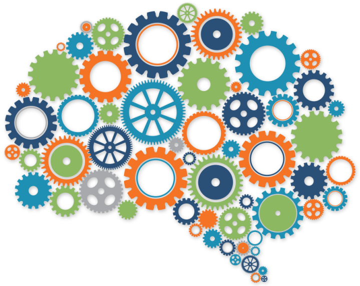


Critical Thinking & Innovation

Jake Chandler

Probabilistic Reasoning, Part 1

LA TROBE UNIVERSITY

Introducing Probability

*In*ductive validity

- Some good reasons don't *guarantee* the truth of a conclusion

Example: Marseille

(P_1) The past ten times that I've been to Marseille, my car has been broken into.

(C) The next time I go to to Marseille, my car will be broken into.

Example: Covid

(P_1) My rapid antigen test came out positive.

(C) I have covid.

*In*ductive validity (ctd)

- Seemingly, these arguments *don't* have intended hidden premises

Example: Covid (Deductive version)

(P_1) My rapid antigen test came out positive.

(P_2) If someone's rapid antigen test comes out positive then they have covid.

(C) I have covid.

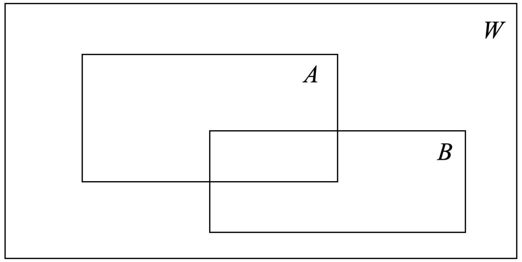
- That would make the argument deductively valid but unsound

*In*ductive validity (ctd)

- What is the relation between premises and conclusion here?
- Plausibly: the premises make the conclusion probable
- These are sometimes called inductively valid (strong, forceful) arguments
- Today: a basic visual introduction to probability
- Note: There's a lively philosophical debate about what exactly "probability" means

Venn diagrams

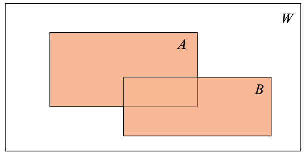
- Sentences can be represented on a Venn diagram, as the sets of possible worlds (situations) in which they'd be true



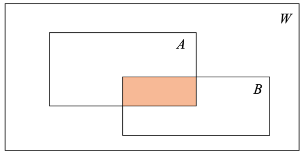
- W is the set of all possible worlds: it represents any tautology

Connectives

- We can represent the construction of complex sentences
- Disjunction ($A \vee B$)

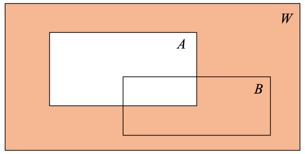


- Conjunction ($A \wedge B$)

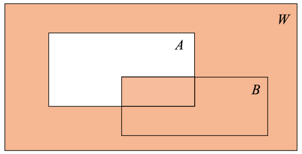


Connectives (ctd)

- Negation ($\neg A$)

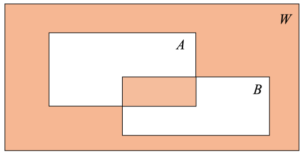


- Conditionals ($A \to B$)



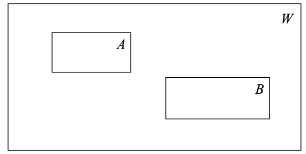
Connectives (ctd)

- Biconditionals ($A \leftrightarrow B$)

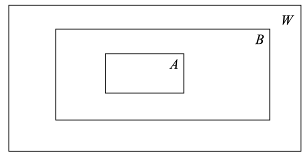


Logical relations

- We can also represent joint inconsistency (when A and B can't be true at the same time, or again when $A \wedge B$ is a contradiction)



…and entailment (when A can't be true without B also being true)



Probability

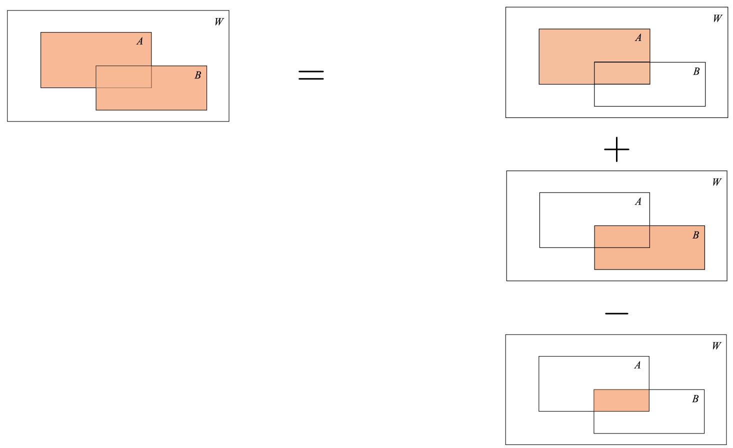
- Finally, we can represent the probability $\Pr(A)$ that a sentence A is true:

  $\Pr(A) =$ the proportion of the area of W (i.e. the total area) that is occupied by A

- This visual understanding is sufficient to recover the various standard rules of probability
- Obvious example: $0 \leq \Pr(A) \leq 1$ (since proportions are all between 0 and 1)

Visualising the rules of probability

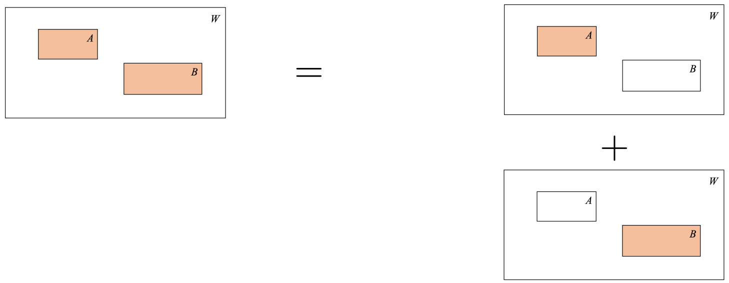
- Example: $\Pr(A \vee B) = \Pr(A) + \Pr(B) - \Pr(A \wedge B)$



Visualising the rules of probability (ctd)

- Example: If A and B are jointly inconsistent (i.e. $A \wedge B$ is a contradiction), then $\Pr(A \vee B) = \Pr(A) + \Pr(B)$

  (special case of previous rule)



Visualising the rules of probability (ctd)

- We use the previous rule all the time!

Example

Chances of drawing a non-face spade from a deck of cards?

Drawing a non-face spade = $1\spadesuit \vee \ldots \vee 10\spadesuit$

These 10 disjuncts are jointly inconsistent and hence by the previous rule:

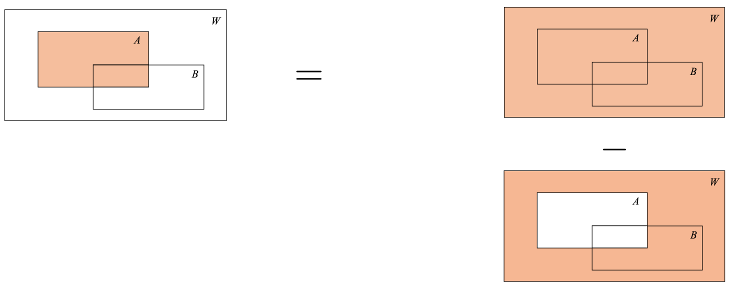
$\Pr(1\spadesuit \vee \ldots \vee 10\spadesuit) = \Pr(1\spadesuit) + \ldots + \Pr(10\spadesuit)$

Since they have the same probability of $\frac{1}{52}$:

$\Pr(1\spadesuit \vee \ldots \vee K\spadesuit) = \frac{10}{52}$

Visualising the rules of probability (ctd)

- Example: $\Pr(A) = 1 - \Pr(\neg A)$



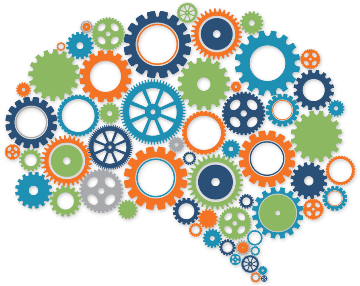
Visualising the rules of probability (ctd)

- Again, we also use this rule frequently!

Example

What are the chances of having a losing ticket (L) in a 100 ticket lottery?

$\Pr(L) = 1 - \Pr(\neg L) = 1 - \frac{1}{100} = 0.99$



Critical Thinking $\wedge$ Innovation

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Probabilistic Reasoning, Part 1

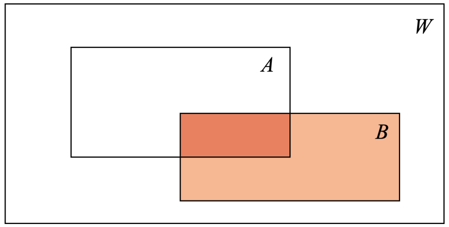
La Trobe University

Conditional probability

Conditional probability

- Key concept: the conditional probability $\Pr(A \mid B)$ of A given B
- Meaning: how confident we should be of A, if we found out that B
- It too can be represented on a Venn diagram:

$\Pr(A \mid B) =$ the proportion of the area of B that is occupied by A

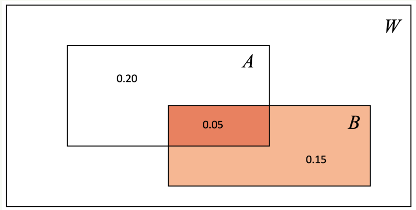


Calculating conditional probability

- Calculating conditional probabilities from unconditional ones:

$$\Pr(A \mid B) = \frac{\Pr(A \wedge B)}{\Pr(B)} \qquad \text{(assuming } \Pr(B) \neq 0\text{)}$$

Example



Here $\Pr(A \mid B) = \dfrac{0.05}{0.20} = 0.25$

Bayes' Theorem

- A useful consequence of the previous equality is Bayes' Theorem:

$$\Pr(H \mid E) = \frac{\Pr(E \mid H)\Pr(H)}{\Pr(E \mid H)\Pr(H) + \Pr(E \mid \neg H)(1 - \Pr(H))}$$

(assuming $\Pr(E) > 0$ and $1 > \Pr(H) > 0$)

- Used to calculate the probability of a hypothesis (H) conditional on the evidence observed (E) (posterior probability)
- We only need the
  - Prior probability: $\Pr(H)$
  - Likelihoods: $\Pr(E \mid H)$ and $\Pr(E \mid \neg H)$
- These are typically known

Bayes' Theorem: Example

Example

Breathalisers never fail to detect a drunk person and falsely show sober drivers to be drunk in only 5% of cases. 1 driver in 1, 000 is driving drunk. A driver is stopped at random and fails the test (tests positive). Can we be confident that he or she is drunk?

E = The driver tests positive (with $\neg E$ = The driver tests negative)
H = The driver is drunk

$\Pr(E \mid H) = 1$ $\quad$ $\Pr(E \mid \neg H) = 0.05$ $\quad$ $\Pr(H) = 0.001$

$$\begin{aligned}\Pr(H \mid E) &= \frac{\Pr(E \mid H)\Pr(H)}{\Pr(E \mid H)\Pr(H) + \Pr(E \mid \neg H)(1 - \Pr(H))} \\ &= \frac{1 \times 0.001}{1 \times 0.001 + 0.05 \times (1 - 0.001)} \approx 0.02\end{aligned}$$

(In)dependence

- E can make H more likely: $\Pr(H \mid E) > \Pr(H)$ (positive dependence)

Example

H = I have covid; E = My rapid test is positive

- E can make H less likely: $\Pr(H \mid E) < \Pr(H)$ (negative dependence)

Example

H = I *don't* have covid; E = My rapid test is positive

(In)dependence (ctd)

- E can make H neither more nor less likely: $\Pr(H \mid E) = \Pr(H)$ (independence)

Example

H = I have covid; E = I was born on an even numbered day of the month

- You can check that these relations are symmetric (e.g. if H depends on E, then E depends on H)

Joint probabilities and independence

- When A and B are independent, it's easy to calculate $\Pr(A \wedge B)$
- Recall:

$$\Pr(A \mid B) = \frac{\Pr(A \wedge B)}{\Pr(B)}, \text{ hence}$$

$$\Pr(A \wedge B) = \Pr(A \mid B) \Pr(B)$$

- If A and B are independent:

$$\Pr(A \wedge B) = \Pr(A) \Pr(B)$$

Example

Chances of getting 2 consecutive tails? Since the tosses are independent: $\Pr(T_1 \wedge T_2) = \Pr(T_1) \times \Pr(T_2) = 1/2 \times 1/ = 1/4$