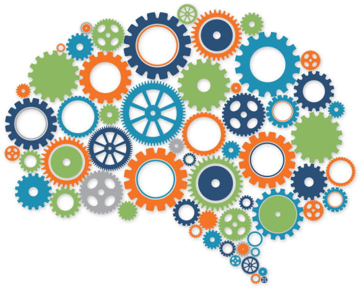


Critical Thinking & Innovation

Jake Chandler

Probabilistic Reasoning, Part 2

LA TROBE UNIVERSITY

Recap of last week

- Cases of seemingly good arguments ("inductively valid" arguments) that are not deductively valid
- The suggestion that the truth of their premises would make the conclusion merely probable
- A very basic introduction to probability:
  - A visual way of thinking about probability as a proportion
  - The concept of something being probable *given* a certain assumption (conditional probability)
  - How to figure out how probable a hypothesis would be given some evidence (Bayes' Theorem)
  - The concepts of dependence and independence

Inductive validity

Conditional probability and inductive validity

- We can now define inductive validity more precisely:

  An inductively valid argument is such the probability of its conclusion being true is high, given the truth of its premises

- More formally:

  An inductively valid argument is such that $\Pr(C \mid P_1 \wedge \ldots \wedge P_n) \geq t$, for some relevant threshold t

- Bayes' Theorem can help assess whether or not this holds
- Inductive soundness is then defined in the obvious way

  An inductively sound argument is an inductively valid argument whose premises are all true

How high is "high"?

- What counts as "high" might depend on context
- Think differences in standards of proof in legal matters:
  - "beyond a reasonable doubt" (criminal cases)
  - "on clear and convincing evidence"
  - "on the preponderance of evidence" or "on the balance of probabilities" (civil cases)
- Important part of context: costs / benefits (of error / success)

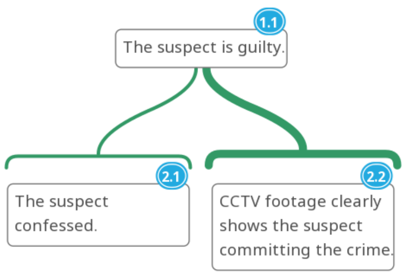
Benefits of the definition

- Assume we say that good arguments can also be inductively valid ones (not just deductively valid)
- Assume we define inductive validity as above
- Then we can explain two key features of good arguments
  (1) Variable strength
  (2) Defeasibility
- With deductive validity alone, we'd have no room for either

Strength of support

- Measuring the strength of inductively valid arguments by $\Pr(C \mid P_1 \wedge \ldots \wedge P_n)$ allows us to explain:

  Variable strength: The degree of support that the premises of an argument provides for the conclusion can differ from one argument to another

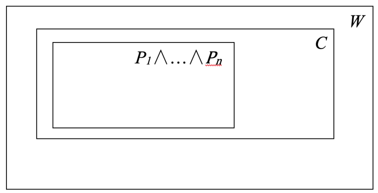


Strength of support and deductive validity

- Deductively valid arguments are then all maximally strong:

  If $P_1 \wedge \ldots \wedge P_n$ entails C, then $\Pr(C \mid P_1 \wedge \ldots \wedge P_n) = 1$

  (Unless the premises are jointly consistent, since the strength is then undefined!)



- $\Pr(C \mid P_1 \wedge \ldots \wedge P_n) =$ proportion of $P_1 \wedge \ldots \wedge P_n$ occupied by C
- But here, C occupies 100% of the area of $P_1 \wedge \ldots \wedge P_n$!

Strength of support and deductive validity (ctd)

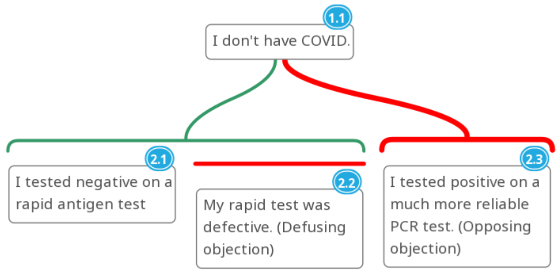
- It follows that :
  - (a) All deductively valid arguments are also inductively valid
  - (b) All deductively valid arguments have the same strength

Defeasibility

- Defining inductively valid arguments as above also explains

  Defeasibility: For some arguments, there is further information given which the truth of the premises would no longer constitute a sufficient reason to believe the conclusion

- This accommodates defusing and opposing objections



Defeasibility and deductive validity

- Arguments that are deductively valid are not defeasible:

  If an argument from $P_1, \ldots, P_n$ to C is deductively valid, so too is an argument from $P_1, \ldots, P_n, D$ to C (for *any* D!)

Example: Still out and about

(P_1) If he was back, then the lights would be on.

(P_2) It is not the case that the lights are on.

(D) ?

(C) It isn't the case that he is back.

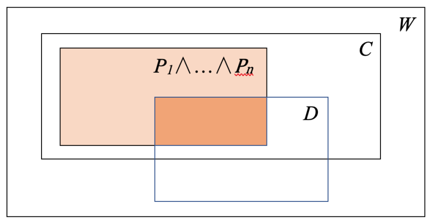
- *Why* are they not defeasible?

Defeasibility and deductive validity (ctd)

- Recall deductive validity:

  All possible situations in which the premises are true are also situations in which the conclusion is true

- But the possible situations in which $P_1, \ldots, P_n$ are true *also include* those in which $P_1, \ldots, P_n$ and D are true!



Defeasibility and conditional probability

- Arguments that are merely inductively valid are defeasible:

  The conditional probability of a conclusion given the premises can fall below the threshold if you add further information

Example: Defective RAT

P = I tested positive on a rapid antigen test

C = I have COVID

D = My antigen test was defective

Let's say for example that t is 0.6

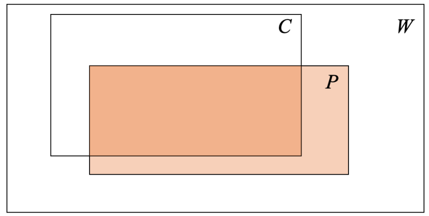
We could plausibly have

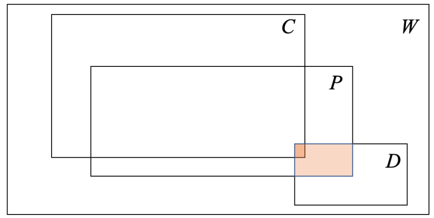
$\Pr(C \mid P) = 0.65$

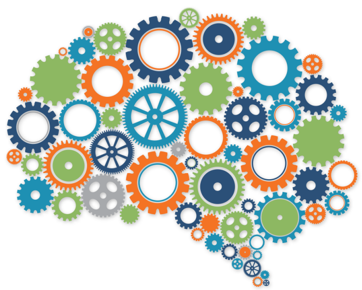
$\Pr(C \mid P \wedge D) = \Pr(C \mid D) = 0.2$ (C and P independent given D)

Defeasibility on a Venn diagram

- Case in which $\Pr(C \mid P)$ is high but $\Pr(C \mid P \wedge D)$ is low:







Critical Thinking & Innovation

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Probabilistic Reasoning, Part 2

La Trobe University

Common Fallacies In Probabilistic Reasoning

Introduction

- Assessing probabilities is *crucial* to assessing arguments but we aren't good at it
- Our breathalyser example showed one common mistake in probabilistic reasoning: the base rate fallacy
- This involved disregarding the relevance of $\Pr(H)$ when calculating $\Pr(H \mid E)$
- Other common errors in probabilistic reasoning include:
  (1) Prosecutor's fallacy
  (2) Conjunction fallacy
  (3) Gambler's fallacy
  (4) Availability fallacies
  (5) Anchoring fallacies

Heuristics

- These are systematic errors, not just random calculation mistakes
- They arguably stem from the use of heuristics, cognitive shortcuts that are
  - faster, easier or require less information, but
  - not as reliable in as many situations, leading to biases
- We'll touch upon 4 heuristics:
  - Likelihood heuristic
  - Representativeness heuristic
  - Availability heuristic
  - Anchoring-and-adjusting heuristic

Dual systems

- Reliance on heuristics is associated with "System 1" in popular dual process models of cognition (Kahneman 2011)

System 1	System 2
subconscious	conscious
automatic	deliberate
evolutionarily primitive	less primitive
fast	slow
effortless	requires effort
for basic tasks:	for higher-order tasks
– recognising faces	– argumentation
– regulating trust	– planning

Overcoming bias

- Although System 1 is the default for many tasks, System 2 can be brought in as a corrective
- To avoid being misled by heuristics we need:
  (i) To be aware of what these are and what situations they are used in
  (ii) To know the correct System 2 reasoning to apply
  (iii) To resist our impulses in those situations and switch to System 2
- I helped you with (ii) last week
- I'll help you with (i) in what follows
- (iii) is your call!

(1) Prosecutor's fallacy

(1) Prosecutor's fallacy

- Question:

  The blood found at a crime scene matches (M) the defendant's and it is 5% probable that a person who is *not* the source (S) of the blood would match by coincidence. How likely is it that this was the defendant's blood?

- Common answer: 95%
- Incorrect!

  This is the figure for $1 - \Pr(M \mid \neg S) = 1 - 0.05$, but we're after $1 - \Pr(\neg S \mid M)$ $(= \Pr(S \mid M))$!

  In general, $\Pr(A \mid B) \neq \Pr(B \mid A)$ (e.g. $A =$ I roll a 2; $B =$ I roll an even number)

- What's going on?!

(1) Prosecutor's fallacy

- Not clear but consider this simple version of Bayes' Theorem

$$\Pr(H \mid E) = \frac{\Pr(E \mid H)\Pr(H)}{\Pr(E)}$$

- Two possibly relevant observations:
  - $\Pr(H \mid E)$ increases with $\Pr(E \mid H)$, holding the rest fixed
  - $\Pr(H \mid E) = \Pr(E \mid H)$ when $\Pr(H) = \Pr(E) = \frac{1}{2}$: initial ignorance about E and H would legitimate the identification
- Might likelihoods be handy but sometimes misleading substitutes for posteriors? (likelihood heuristic)

References

- Kahneman, D. (2011). *Thinking, fast and slow*. Farrar, Straus and Giroux.