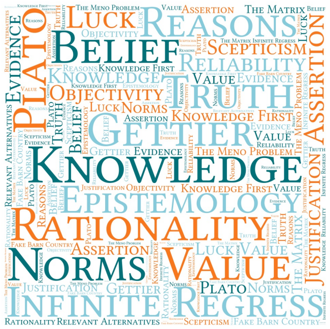


Knowledge, Information & Society

Jake Chandler

Probability: Basics

Introduction

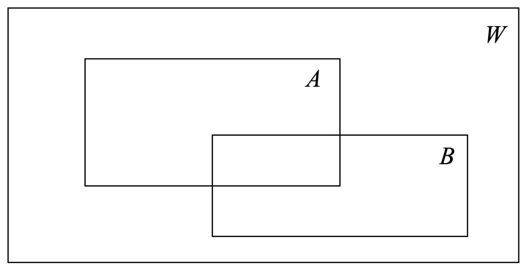
- This is a largely informal course, but there are some unavoidable technical notions
- What follows is *optional* but *strongly recommended* minimal background
- A fair amount of the literature that we touch on can require a little more
- For further reading, see Hájek & Hitchcock (2017)

Propositions

- Consider the set W of all the ways the world could have been (the set of all possible worlds)
- To keep things simple: assume W is finite
- We can identify a proposition (aka "event") with a set of worlds: the set of worlds in which it is true
- W is the proposition that *must* be true (tautology)
- Similarly the empty set $\varnothing$ is the proposition that *cannot* be true (contradiction)

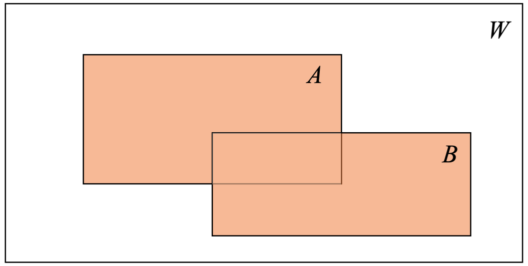
Venn diagrams

- Propositions can be represented on a Venn diagram



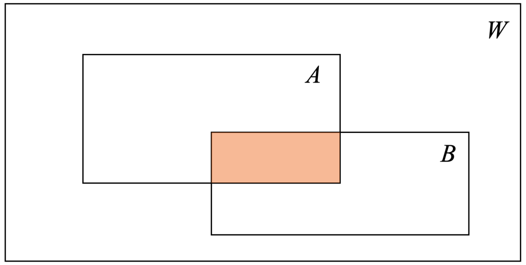
Building sentences and sets

- Propositions can be "built" from other propositions
- Union (think disjunction: "or"): $A \cup B$



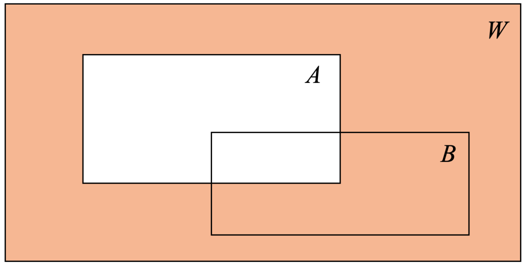
Building sentences and sets

- Intersection (think conjunction: "and"): $A \cap B$



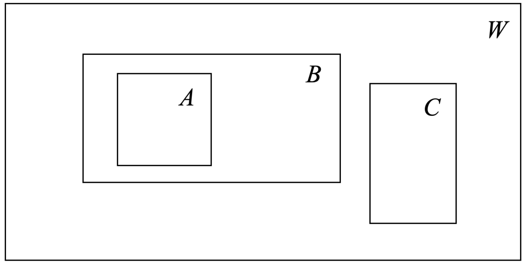
Building sentences and sets

- Complementation (think negation: "not"): $\overline{A}$



Subsets

- A proposition A entails a proposition B when A a subset of (is contained in) B ($A \subseteq B$)
- Two propositions are jointly inconsistent, when they have an empty intersection (aka the propositions are "disjoint" or "non-overlapping")



Probability

- A probability function takes a proposition and tells us how likely that proposition is to be true
- There's a lively philosophical debate about what exactly "probability" means and whether it is ambiguous
- Candidate meanings:
  - Rational degrees of confidence
  - Frequencies (actual or hypothetical)
- We'll sidestep this here. For further reading, see Hájek (2019)

Probability

- We use $\Pr(A)$ to denote the probability that A is true
- All the rules of probability can be derived from:
  (1) $\Pr(A) \geq 0$
  (2) $\Pr(W) = 1$
  (3) If $A \cap B = \varnothing$, then $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
- Examples:

  $\Pr(A) = 1 - \Pr(\overline{A})$

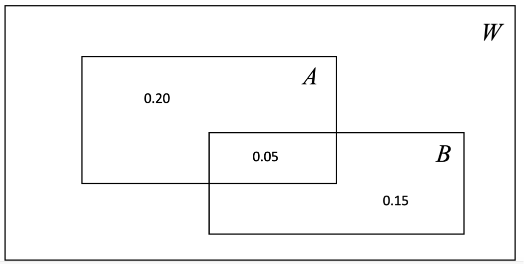
  $\Pr(\varnothing) = 0$

  $0 \leq \Pr(A) \leq 1$

  If $A \subseteq B$, then $\Pr(A) \leq \Pr(B)$

Probability on Venn Diagrams

- You could think of probabilities as proportions of an area of size 1

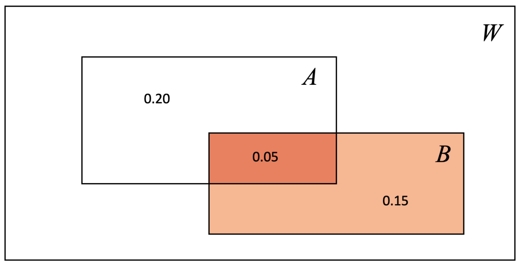


Conditional probability

- Important notion: the conditional probability of A given B

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} \qquad \text{(assuming } \Pr(B) \neq 0\text{)}$$

(The proportion of the area of B that is also occupied by A)



Here $\Pr(A \mid B) = \dfrac{0.05}{0.20} = 0.25$

(In)dependence

- The truth of B can make that of A
  - more likely: $\Pr(A \mid B) > \Pr(A)$ (positive dependence)

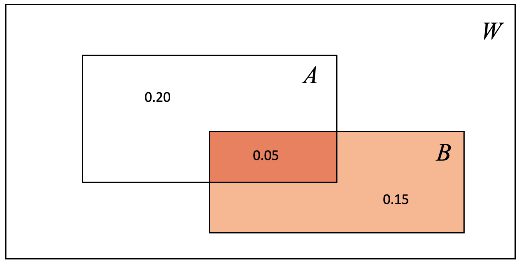
    Example: My having covid and my receiving a positive result on my RA test
  - less likely: $\Pr(A \mid B) < \Pr(A)$ (negative dependence)

    Example: My *not* having covid and my receiving a positive result on my RA test
  - neither one not the other: $\Pr(A \mid B) = \Pr(A)$ (independence)

    Example: My having covid and my having been born on an even numbered day of the month
- You can check that these relations are symmetric (e.g. if A depends on B, then B depends on A)

(In)dependence: Example

- In our example, A and B were independent



$$\Pr(A \mid B) = \frac{0.05}{0.20} = 0.25 = \Pr(A)$$

Independence: Alternative formulation

- An equivalent version of independence

  $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
- Example 1:

  Chance of a 6 on a dice roll = $\frac{1}{6}$

  Chance of two consecutive 6s = $\frac{1}{6} \times \frac{1}{6}$
- Example 2 (R. v. Clark):

  Chance of SIDS = $\frac{1}{8{,}543}$

  Chance of SIDS in two siblings $\neq \frac{1}{8{,}543} \times \frac{1}{8{,}543} = \frac{1}{73M}$!

  SIDS in the one sibling raises the chance of SIDS in the other!

(In)dependence for n propositions

- These notions of (in)dependence generalise to n propositions
- For example, for 3 propositions A, B and C, independence is given by

  (i) $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$,

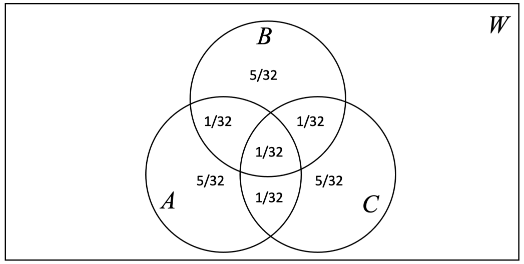
  (ii) $\Pr(A \cap C) = \Pr(A) \times \Pr(C)$,

  (ii) $\Pr(B \cap C) = \Pr(B) \times \Pr(C)$, and

  (iv) $\Pr(A \cap B \cap C) = \Pr(A) \times \Pr(B) \times \Pr(C)$

(In)dependence for *n* propositions (ctd)

- Pairwise independence isn't enough: (i)–(iii) don't entail (iv)!



$\Pr(A \cap B) = 1/16 = 1/4 \times 1/4 = \Pr(A) \times \Pr(B)$, etc.

$\Pr(A \cap B \cap C) = 1/32 \neq$
$1/64 = 1/4 \times 1/4 \times 1/4 = \Pr(A) \times \Pr(B) \times \Pr(C)$

References

- Hájek, A. 2019. Interpretations of Probability. *The Stanford Encyclopedia of Philosophy (Fall 2019 Edition)*, E. N. Zalta (ed.).
- Hájek, A. & C. Hitchcock 2017. Probability for Everyone–Even Philosophers. *The Oxford Handbook of Probability and Philosophy*. A. Hájek & C. Hitchcock (eds.). Oxford University Press.