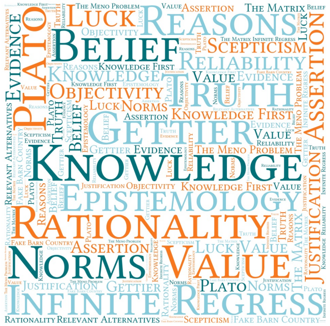

Knowledge, Individuals & Society

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Rational belief
& practical interests: Pascal II

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Pascal's Argument from Dominance

- The relevant passage:

  'You have two things to lose, the true and the good; and two things to stake, your reason and your will, your knowledge and your happiness; and your nature has two things to shun, error and misery...Let us weigh the gain and the loss in wagering that God is...*If you gain, you gain all; if you lose, you lose nothing.* Wager, then, without hesitation that He is.'

Pascal's Argument from Dominance (ctd)

- The decision table:

	God exists (G)	God doesn't exist ($\neg G$)
Wager for (W_G)	a	b
Wager against ($W_{\neg G}$)	c	d

- Formally, here are Pascal's claim regarding payoffs:
  - $a > c$
  - $b \geq d$ ('If you lose, you lose nothing')
- Jargon: wagering for 'weakly dominates' wagering against
- We'll also assume that he takes the states to be independent of the acts: without this, the argument can fail (more on this later)

Weak dominance and fervent atheism

- Technically, however, weak dominance isn't *by itself* sufficient to secure rationality of wagering for (Hajek 2012)
- Assume, say, that $b = d$ (this is consistent with weak dominance): in case God doesn't exist, it makes no difference to me whether I wager for or against him
- Assume further that my evidence entails that I should be 100% certain that God doesn't exist ($\Pr(G) = 0$)
- The fact that $a > c$ becomes irrelevant: I should be indifferent between wagering for and wagering against
- So Pascal does indeed need to *at least* assume that our evidence does not entail that $\Pr(G) = 0$ (i.e. doesn't support $\neg G$)

Risk vs Uncertainty

- Is the assumption sufficient?
- The following *would* certainly be sufficient:
  (a) Our evidence entails that $\Pr(G) > 0$
- It is easy to show that, given (a), weak dominance suffices for W_G to be the unique act that maximises expected utility
- But our assumption that
  (b) Our evidence doesn't entail that $\Pr(G) = 0$

  falls short of this: our evidence may be insufficient to sensibly assign *any* probability to G at all

Risk vs Uncertainty (ctd)

- If we only assume (b), the validity of Pascal's argument hinges on whether we opt for Indifference or Maximin
  - Indifference: The argument goes through
  - Maximin: The argument doesn't go through. Counterexample:

	G	$\neg G$
W_G	5	2
$W_{\neg G}$	2	2

  W_G and $W_{\neg G}$ have the same worst outcomes (2): both are equally choice-worthy!

  So weak domination isn't sufficient to *mandate* choice

Taking stock

- So Pascal's Argument from Dominance goes through on the assumptions that
  - W_G weakly dominates $W_{\neg G}$
  - Our evidence entails that $\Pr(G) > 0$
  - W_G / $W_{\neg G}$ are independent of G / $\neg G$
- We have seen the reasons for the second requirement ("fervent atheism" and decisions under uncertainty)
- But why the third? (for full discussion, see Hajek 2012)

Act/state dependence

- Consider:

	Offer	¬Offer
¬Apply	$a = 5$	$b = 1$
Apply	$c = 4$	$d = 0$

- Not applying (strictly) dominates applying, but I should apply
- This is because the states of nature are not independent of the acts: if I don't apply, I am unlikely to get an offer!
- Let's see how dependence can mean that strictly dominating acts can fail to be choice-worthy…

Act/state dependence (ctd)

	Offer	¬Offer
¬Apply	$a = 5$	$b = 1$
Apply	$c = 4$	$d = 0$

- Here:

$$\mathrm{EV}(\neg A) = 5 \times \Pr(O \mid \neg A) + 1 \times \Pr(\neg O \mid \neg A)$$
$$\mathrm{EV}(A) = 4 \times \Pr(O \mid A) + 0 \times \Pr(\neg O \mid A)$$

- Depending on the probabilities of states given acts, we could have $\mathrm{EV}(A) > \mathrm{EV}(\neg A)$, *even though* $\neg A$ strictly dominates A.
- Example:

$$\mathrm{EV}(\neg A) = 5 \times 0.01 + 1 \times 0.99 = 1.04 \; <$$
$$\mathrm{EV}(A) = 4 \times 0.8 + 0 \times 0.2 = 3.2$$

References

- Hajek, A. 2012. Blaise and Bayes. In J. Chandler and V. Harrison (eds.), *Probability in the Philosophy of Religion*, Oxford University Press, Oxford UK, pp. 167–186.

Act/state dependence (ctd)

- State O is independent of act A iff
  - (i) $\Pr(O \mid A) = \Pr(O)$ or equivalently
  - (ii) $\Pr(O \mid A) = \Pr(O \mid \neg A)$
- (ii) failed in the numerical example:

$$\Pr(\neg O \mid \neg A) = 0.99 > \Pr(\neg O \mid A) = 0.2$$

- But if independence held true, then, by (i), we could write

$$\mathrm{EV}(A) = a \times \Pr(O) + b \times \Pr(\neg O)$$
$$\mathrm{EV}(\neg A) = c \times \Pr(O) + d \times \Pr(\neg O)$$

- It is easy to see that either of the following then suffice for $\mathrm{EV}(\neg A) > \mathrm{EV}(A)$
  - (i) weak dominance of $\neg A$ *plus* $0 < \Pr(O) < 1$
  - (ii) strict dominance of $\neg A$