

Relaxing OUTPUT CONSISTENCY (ctd)

- Inputs *violating* UNIDIMENSIONAL ALIGNMENT (Discursive Dilemma):

	1	2	3	Maj
E	T	T	F	T
$E \rightarrow T$	T	F	T	T
T	T	F	F	F

- Exercise: draw up the 6 ($3 \times 2 \times 1$) possible ways of ordering the voters and verify the violation

Relaxing OUTPUT CONSISTENCY (ctd)

- There is in fact a more general condition on inputs that is sufficient for OUTPUT CONSISTENCY: **VALUE-RESTRICTEDNESS** (Dietrich & List 2010)
- Question: right, but what *proportion* of all inputs do these consistency-guaranteeing inputs correspond to?
- No general answer, to my knowledge
- List (2005) calculates the probability of inconsistency given n in *the Discursive Dilemma*, under the assumption that all possible relative frequencies of sets of judgments are equiprobable ('**impartial anonymous culture**')

Relaxing OUTPUT CONSISTENCY (ctd)

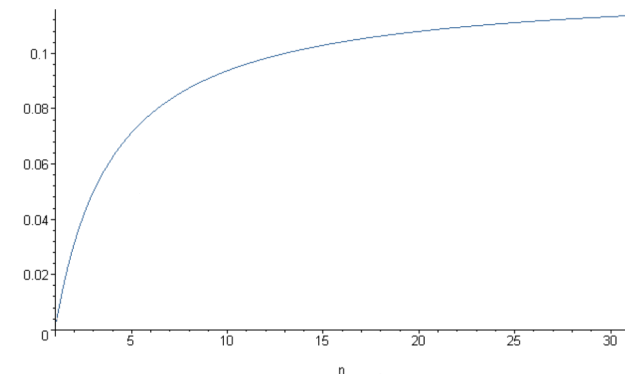
- Inputs *satisfying* UNIDIMENSIONAL ALIGNMENT:

	1	2	3	Maj
E	T	T	F	T
$E \rightarrow T$	T	F	F	F
T	T	F	F	F

- Explanation, for when n is odd:
 - For any proposition, the majority votes in favour iff the median voter (here: #2) does
(Why so? Because on either the left or the right of the median voter, we will have enough voters agreeing with her to make up a majority if we add her vote to theirs)
 - The group judgment will correspond to the (consistent and complete) judgment of the median voter

Relaxing OUTPUT CONSISTENCY (ctd)

- The probability looks reasonably high:



Premise/Conclusion-based voting

SYSTEMATICITY

- Popular way of dropping SYSTEMATICITY: majority voting on a **restricted subset** of the agenda
- Sometimes, it makes sense to divide the agenda into
 - a set of **premises** (e.g. $\{E, E \rightarrow T\}$), that entail
 - a set of **conclusions** (e.g. $\{T\}$)

so that, ultimately, we only really care about the conclusions

- We then have the options:
 - **Premise-based** majority voting (voting on just the premises, then taking the stance on the conclusions that follows)
 - **Conclusion-based** majority (voting on just the conclusions and suspending judgment on the premises)

Premise/Conclusion-based voting (ctd)

	E	$E \rightarrow T$	T
Person 1	T	T	T
Person 2	T	F	F
Person 3	F	T	F
Majority	T	T	F
Premise-based	T	T	T
Conclusion-based	-	-	F

Premise/Conclusion-based voting(ctd)

- Clearly, both methods satisfy ANONYMITY
- Given ANONYMITY, SYSTEMATICITY amounts to

The group judgment on all proposition is determined in the same way by the **frequencies** of votes on that proposition (e.g. true if freq. =0.75 and false if not)
- Both methods violate this, since $f(E) = f(\neg T) = \frac{2}{3}$ but:
 - *Conclusion-based*: E isn't believed but $\neg T$ is
 - *Premise-based*: E is believed but $\neg T$ is not
- In fact, these methods violate *both* parts of SYSTEMATICITY (I'll leave that as an exercise for you to prove)

Premise/Conclusion-based voting (ctd)

- OUTPUT CONSISTENCY is satisfied in both cases:
 - *Premise-based*: if the premises are logically independent
 - *Conclusion-based*: if the conclusions are logically independent
- Ok, but how likely are these procedures to yield true judgments on the entire agenda? See Hartmann & Sprenger (2012)

Voting on atoms

- What if we *can't* divvy things up into premises and conclusions?
- One option:
 - Majority vote on the **atomic propositions**, i.e. logically simple propositions associated with the agenda (the letters inside the formulae)
- The truth values of these atomic sentences then logically determine the truth value of all sentences on the agenda
Example: $E \rightarrow T$ is true iff either E is false or T is true
- Note: this only works if the atomic propositions *are on the agenda*; if they aren't, a more general approach, the **distance-based** approach can be used

Voting on atoms (ctd)

	E	$E \rightarrow T$	T
Person 1	T	T	T
Person 2	T	F	F
Person 3	F	T	F
Majority	T	T	F
Premise-based	T	T	T
Conclusion-based	-	-	F
Atom-based	T	F	F

Voting on atoms (ctd)

- Majority voting on atoms obviously satisfies ANONYMITY
- It's then easy to see that it violates SYSTEMATICITY (NEUTRALITY):
 - 2/3 of the voters judged $E \rightarrow T$ to be true, and 2/3 judged T to be false, but the group judged both to be false
- It also guarantees OUTPUT CONSISTENCY, since atomic propositions are logically independent
- How likely is this method to yield true judgments on the entire agenda? See again Hartmann & Sprenger (2012) for a discussion and comparison with premise- and conclusion-based procedures

Voting on atoms (ctd)

- One noteworthy thing about this procedure is that it violates

UNANIMITY: The group judgment should respect unanimity

	P	Q	R	$P \vee Q \vee R$
Person 1	T	F	F	T
Person 2	F	T	F	T
Person 3	F	F	T	T
Atom-based	F	F	F	F

- Is that bad?

OUTPUT DECISIVENESS

Relaxing OUTPUT DECISIVENESS

- Supermajority and Unanimity violate OUTPUT DECISIVENESS (as does Majority, for even n)
- List & Pettit's theorem doesn't predict trouble here...but the Discursive Dilemma highlights problems with these!!
- Could there be a general result that explains what the problem is?
- We've seen that ANONYMITY and SYSTEMATICITY give us the requirement that group judgment wrt to a proposition is determined by the frequencies of votes on that judgment
- But this doesn't tell us *anything* about *how* it is determined

Relaxing OUTPUT DECISIVENESS (ctd)

- One last condition, then:
If the group judges a proposition to be true, it would have still judged that proposition to be true had one of the voters who voted it to be false voted it to be true instead
- Given ANONYMITY and SYSTEMATICITY, this amounts to
MONOTONICITY: If (i) a proposition P would be judged true by the group on the basis of a frequency of votes F_1 and (ii) P^* were to receive a frequency of votes $F_2 \geq F_1$, then (iii) P^* would be judged true by the group
- Does this not look familiar?

References

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