Introduction to Logic

Jake Chandler

Validity & Proof + Sentential Form

LA TROBE UNIVERSITY

Last week

- Why we should study logic
- The concept of an argument, of a premise and of a conclusion
- How to represent an argument in standard form
- Common issues in trying to reconstruct a real world argument

Deduction: First Steps

Deductively valid arguments

- One key feature of a good argument:

  The premises support the conclusion

- The gold standard for support:

  Deductive validity: The truth of the premises would (100%) guarantee that of the conclusion

  (We also say that the premises entail the conclusion.)

Square

(1) The shape has four equal sides

(2) The shape's inner angles are ninety degrees

---

(3) The shape is a square

Deductive *in*validity

- If an argument is deductively invalid, it could have possible for premises to be true but the conclusion false

Square #2

(1) The shape has four equal sides

---

(2) The shape is a square

(Relevant possibility: the shape is a diamond, so that (1) is true but (2) false)

- Let's test our judgments about deductive validity…

Deductive validity vs invalidity: examples

Treats

(1) Isaac either got a cookie or got an ice-cream.
(2) Isaac got a cookie.

---

(3) Isaac didn't get an ice-cream.

Steroids

(1) If steroids were taken, the test will come out positive.
(2) Sam's test came out positive.

---

(3) Sam took steroids.

Deductive validity vs invalidity: examples (ctd.)

Penknife

(1) If Billy's dad found his penknife, he's in deep trouble.
(2) Billiy's dad didn't find his penknife.

---

(3) Billy isn't in deep trouble.

Cluedo

(1) Either Professor Plum did it, or Colonel Mustard did.
(2) Professor Plum didn't do it.

---

(3) Colonel Mustard did it.

Deductive validity vs invalidity: examples (ctd.)

Prison

(1) Some residents of the prison went to the zoo.
(2) All prisoners are residents of the prison.

---

(3) Some prisoners went to the zoo.

Hooligans

(1) Some football fans are hooligans.
(2) All hooligans are bad-tempered.

---

(3) Some football fans are bad-tempered.

Deductive validity vs invalidity: examples (ctd.)

- What about this one, which we saw earlier?

Doors

(1) The sign on the red door says that the prize is behind the red door.

(2) The signs on both the blue door and the green door say that the prize isn't behind the blue door.

(3) At least one of the signs states a falsehood and at least one of the signs states a truth.

---

(4) The prize is behind the green door.

Deductive validity vs invalidity: examples (ctd.)

- *Doors is* in fact deductively valid.
- This might *look* a little difficult to figure out…
- But we'll see later on that, given the right tools, it's actually really easy.

Tautologies and contradictions

- Two important concepts related to validity:

  Tautology: A sentence that had to be true (couldn't possibly have been false)

  Contradiction: A sentence that had to be false (couldn't possibly have been true)

- A sentence that is not a contradiction is called satisfiable
- A sentence that is neither a contradiction nor a tautology is called contingent
- An argument is valid if and only if:

  It is a tautology that, if its premises are true, then its conclusion is also true

  It is a contradiction that both its premises are true and its conclusion false.

*In*ductive validity

- It doesn't follow from the fact that an argument isn't deductively valid, that it isn't a good argument!
- There are good reasons that fall short of *guaranteeing* truth.

Marseille

(1) The past ten times that I've been to Marseille, my car has been broken into.

---

(2) The next time I go to to Marseille, my car will be broken into.

*In*ductive validity (ctd)

Covid

(1) My rapid antigen test came out positive

---

(2) I have covid.

- Above, (1) seems to provide a good enough reason to expect (2), but doesn't *guarantee* its truth (I might be lucky)
- These arguments are inductively valid
- We set these aside here: henceforth "valid" = "deductively valid"

Validity & soundness

- Premise support for the conclusion is only one of two requirements for an argument's being good
- There is more to a good deductive argument than mere validity:

Intergalactic Renegades

(1) Jake teaches logic

(2) If you teach logic, then you're an Intergalactic Renegade

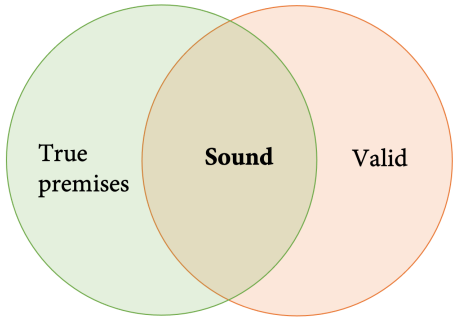
---

(3) Jake is an Intergalactic Renegade

- Valid: the truth of (1) and (2) would guarantee that of (3)
- But unpersuasive: (2) is clearly false

Validity & soundness (ctd)

- A sound argument is an argument (i) that is valid *and* (ii) whose premises are all true



- So every sound argument has a true conclusion. (But *not* all arguments with true conclusions are sound!!!)

Validity & soundness (ctd)

- If you reject the conclusion of an argument, you have a duty to show that the latter is unsound, by either:
  - (i) showing that the argument is invalid, or
  - (ii) casting doubts on the truth of the premises.
- In this subject, we will *only* focus on validity: assessing the truth of the premises or an argument is not the job of the logician.

Introduction to Logic

Jake Chandler

Validity & Proof + Sentential Form

LA TROBE UNIVERSITY

A coincidence?

Cluedo

(1) Either Professor Plum did it, or Colonel Mustard did.
(2) It isn't the case that Professor Plum did it.

---

(3) Colonel Mustard did it.

Keys

(1) Either my car keys are at home, or they are in the office.
(2) It isn't the case that my car keys are at home.

---

(3) My car keys are in the office.

A coincidence? (ctd)

Still out and about

(1) If he was back, then the lights would be on.
(2) It is not the case that the lights are on.

---

(3) It isn't the case that he is back.

Departure

(1) If he had been planning on leaving, then he would have contacted Sam.
(2) It isn't the case that he contacted Sam.

---

(3) It isn't the case that he was planning on leaving.

A coincidence? (ctd)

Hiding

(1) He either stayed in Damascus or went to Aleppo.
(2) If he stayed in Damascus, then he will have been arrested.
(3) If he went to Aleppo, then he will have been murdered.

---

(4) He will have been arrested or murdered.

Nightclub

(1) Either it'll be crowded or it'll be empty.
(2) If it's crowded, then it'll be too hot.
(3) If it's empty, it'll be boring.

---

(4) It'll either be boring or too hot.

Formal validity

- All these arguments are valid
- But their validity doesn't seem to depend on *what they are about*
- It looks like it's just the form (pattern, structure,…) that matters.

  (Note: whether or not the arguments are *sound*, however, will also depend on subject matter!)
- You can't obtain true premises but a false conclusion by changing the content but preserving the form!

A note on formal validity

- An argument may well be valid even though it isn't *formally* valid, i.e. instantiates no valid form at all!
- A case in point:

Bachelor

(1) Ben is a bachelor

---

(2) Ben is unmarried.

Formal validity (ctd.)

- Argument form can be described at two levels of detail
  (i) Sentential form (less detailed)
  (ii) Sub-sentential form (more detailed)
- Our examples above are valid simply because of their sentential forms

Introduction to Logic

Jake Chandler

Validity & Proof + Sentential Form

LA TROBE UNIVERSITY

Sentences and sub-sentences

- A complex sentence is a sentence built up from other sentences
- These sentential building blocks, which can themselves be complex, are known as sub-sentences
- Not all sentences are complex, however
- An atomic sentence is a sentence that has no further component sub-sentences

Atomic vs complex sentences

Atomic vs complex sentences

Atomic sentences:

It'll be crowded.

He stayed in Damascus.

The lights are on.

Complex sentences (sub-sentences in italics):

Either *it'll be crowded* or *it'll be empty*.

If *he stayed in Damascus*, then *he will have been arrested*.

It is not the case that *the lights are on*.

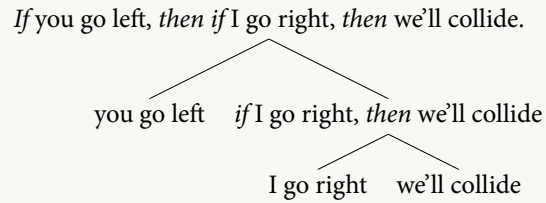
Connectives

- In the examples we have just seen, some parts of the complex sentences were not sub-sentences.
- These elements are known as sentential connectives.
- They take one or more sentences as an input and produce a complex sentence as an output.
- There are *many* connectives, but logicians focus on these "standard" ones:
  - …and…
  - …or…
  - if…, then…
  - …if and only if…
  - It is not the case that…
- Later: *why* the standard connectives are taken to be special

Parse trees

- A parse tree shows how complex sentences are built up from atomic ones

A parse tree



- The main connective of a complex sentence is the last connective used in constructing it

Types of complex sentences

- We give special names to certain types of sentences based on their main connective.

Main Connective	Sentence name	Input name
…and…	conjunction	conjuncts
…or…	disjunction	disjuncts
if…, then…	conditional	antecendent / consequent
…if and only if…	biconditional	left-hand term / right-hand term
it is not the case that…	negation	negand

Rewriting complex sentences

- Important step in logic:

  rewrite English sentences in an equivalent way, using only standard connectives, to "bring out" the sentential structure

- What counts as "equivalent"? Tricky.
- At the very least, the sentences must be true in exactly the same situations
- But we don't require them to have the exact same connotations or meaning, in the broad sense
- In what follows: a non-exhaustive review of some difficulties with paraphrasing

Conjunctions

- One issue: equivalent non-standard variants
- Typically connectives suggesting an additional notion of contrast

Equivalent variants of "and"

It's raining *but* I am going to the beach
It's raining *yet* I am going to the beach
Although it's raining, I am going to the beach
⇒ It's raining *and* I am going to the beach

Moses is rich, *whereas* Gebril is poor.
⇒ Moses is rich *and* Gebril is poor.

Conjunctions (ctd)

- Another issue: disguised conjunctions

Disguised conjunctions

Both my aunts are annoying.
⇒ My first aunt is annoying and my second aunt is annoying.

Gio is a restaurant owner with a taste for luxury watches.
⇒ Gio is a restaurant owner and Gio has a taste for luxury watches.

Conjunctions (ctd)

Disguised conjunctions (ctd)

Sylvia and Sophie like cheesecake.
⇒ Sylvia likes cheesecake and Sophie likes cheesecake.

Sylvia and Sophie got married [to each other].
$\not\Rightarrow$ Sylvia got married and Sophie got married.

Sam and Pam are sisters [of one another].
$\not\Rightarrow$ Sam is a sister and Pam is a sister.

Conjunctions (ctd)

- Finally: "and" used as a conditional

"and" used as a conditional

One more mistake and he'll be fired.
$\not\Rightarrow$ He'll make one more mistake and he'll be fired.
⇒ If he makes one more mistake, then he'll be fired.

Disjunctions

- Potential issue: disguised *dis*junctions

Disguised disjunctions

George is stupid or deluded.

⇒ George is stupid or George is deluded.

Disjunctions (ctd)

- Another issue: exclusive disjunction
- It is often the case that when people say "P or Q", then really mean "P or Q but not both"

"and" used as a conditional

I told you: You can either go to the zoo or go to the waterpark. You chose to go to the zoo. Stop complaining!

⇒ I told you: You can either go to the zoo or go to the waterpark, *but not to both*.

Conditionals

- Issue: equivalent non-standard variants

Non-standard variants of conditionals

If Hewan will be there, Faisal will be there.

Faisal will be there *if* Hewan will be there.

Hewan will be there *only if* Faisal will be there.

Hewan will *not* be there *unless* Faisal will be there.

⇒ *If* Hewan will be there, *then* Faisal will be there.

If you have something interesting to say, speak up; *otherwise* don't say a word.

⇒ If you have something interesting to say, speak up; *if it is not the case that* you have something interesting to say, *then* don't say a word.

Conditionals (ctd)

Non-standard variants of conditionals (ctd)

One more mistake and he'll be fired.

⇒ If he makes one more mistake, then he'll be fired.

Negations

- Issue: equivalent non-standard variants

Non-standard variants of negation

Not all Belgian administrators are very cheerful.

$\Rightarrow$ It is not the case that all Belgian administrators are very cheerful.

The exam wasn't hard.

$\Rightarrow$ It is not the case that the exam was hard.