Introduction to Logic

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Beyond Truth and Falsity, Pt. 1

LA TROBE UNIVERSITY

Last week

- Tableau methods for predicate logic
- Enriching predicate logic with a symbol "=" for identity
- Tableau methods for predicate logic with identity

The case for gaps

Introduction

- We've assumed bivalence: sentences are either true (1) or false (0)
- Some argue for trivalence: some sentences are neither true nor false (#)
- In those cases: truth value gaps
- Over the next two weeks:
  - different reasons to claim gaps exist
  - various associated proposals regarding handling validity and determining the truth values of complex sentences
  - corresponding sound and complete tableau-based proof systems

Introduction

- The following types of sentences have been argued to be associated with truth value gaps
  (i) Sentences involving definite descriptions
  (ii) Conditionals
  (iii) Universal generalisations
  (iv) Sentences involving aspectual or factive verbs
  (v) Sentences regarding future contingents
  (vi) Sentences involving borderline cases
- The *way* in which these sentences are "gappy" differs however:
  - (i)-(iv): Gappy because they are "defective"
  - (v)-(vi): Gappy because their value is "indeterminate"

(i) Definite descriptions

The King of France

(1) 'The king of France is bald.' (France currently has no king)

- This is clearly not true.
- But neither is:

  (1') 'It is not the case that the king of France is bald.'
- If (1) and (1') seem untrue, doesn't that mean that bivalence fails?
- Classically: $\models \varphi \vee \sim \varphi$
- Strawson: both (1) and (1') seem untrue because they are neither true nor false

(i) Definite descriptions: Russell

- Russell: both (1) and (1') seem untrue because they are false
- He suggests (1) is equivalent to

  'There exists exactly one King of France and that person is bald'
- In our language $\mathscr{L}_{PI}$:

  $(\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& Bx)$
- But what about (1')?
- Don't we then have *that* translated as:

  $\sim (\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& Bx)$
- If so, doesn't he have to say that (1') is *true*?!

(i) Definite descriptions: Russell (ctd)

- Russell: we add a negation inside the scope of $(\exists x)$

  $(\exists x)(Kx \& (\forall y)(Ky \supset y = x) \& \sim Bx)$
- The translation of (1') is *not* the negation of the translation of (1)
- He would offer a similar translation of

  (1") 'It is false that the king of France is bald.'
- No failure of bivalence according to him
- Response: sure, that would save bivalence, but

  What *independent* motivation is there for this translation?

  Why does the negation not figure as the first symbol?

  What English sentence *would* be translated in such a way that negation did appear as the first symbol?

(ii) Conditionals

Dead

(2) 'If I am dead, then my mother is happy.'

- Some would feel strongly that (2) is neither true nor false (most famously De Finetti).
- In support, note the typical response to (2):

  'But you are not dead...'

  rather than:

  'Of course that's true: you're not dead!'

(ii) Conditionals (ctd.)

- Classical view: (2) is true by virtue of the following table

φ	ψ	$\varphi \supset \psi$
1	1	1
1	0	0
0	1	1
0	0	1

- 'De Finnettian' view: (2) is neither true nor false, by virtue of the following alternative partial* table

φ	ψ	$\varphi \supset \psi$
1	1	1
1	0	0
0	1	#
0	0	#

* "Partial" since we need to allow for #-values for φ and ψ

(iii) Universal generalisations

Flying Donkeys

(3) All flying donkeys come home to roost.

- True? False?
- As we've seen:
  - Aristotelian logic: '*false*, as there are no flying donkeys!'
  - Predicate logic: '*true*, as there are no flying donkeys!'
- Strawson (1952, p. 174): '*neither true nor false*, as there are no flying donkeys!'

(iii) Universal generalisations (ctd)

- On our partial trivalent table for $\supset$:

  If $(\forall x)(\sim Dx)$ has the value 1, then every instance of $(\forall x)(Dx \supset Rx)$ has the value #
- Plausibly, if this is the case, then we should also have $v((\forall x)(Dx \supset Rx)) = \#$
- Strawson's view falls out of the the trivalent approach to $\supset$

(iv) Aspectual and factive verbs

Elvis/Arithmetic

(4) 'Elvis Presley has stopped emailing me.'

(5) 'John knows that 2 + 2 = 5.'

- True? False?
- Many philosophers & linguists say: 'neither'.

Presuppositions

- The sentences in (iv) are often said to carry presuppositions,
- When P presupposes Q ($P >> Q$), P entails Q and, when Q is false, asserting P is conversationally inappropriate

Presuppositions

(4) 'Elvis Presley has stopped emailing me.'
>> Elvis Presley used to email me

(5) 'John knows that 2 + 2 = 5.'
>> 2 + 2 = 5

Presuppositions: Test #1

- There exist a number of 'tests' for the presence of presuppositions
- The "Hey, wait a minute" test (von Fintel):

  When a sentence P carries a false presupposition Q, it is appropriate to say "Hey, wait a minute: but it isn't true that Q!" but not appropriate to say this in relation to other implications of P.

"Hey, wait a minute" test

'John knows that $2 + 2 = 5$.'

'Hey, wait a minute: but it isn't true that $2 + 2 = 5$!'

'Hey, wait a minute: but it isn't true that he knows that!'

Presuppositions: Test #2

- Projection test:

  The implications associated with presuppositions survive embedding the original sentence in certain more complex constructions (e.g. as negands, antecedents of conditionals or as complements of attitude verbs)

Projection test: negation

'It's not the case that Elvis Presley has stopped emailing me.'

'It's not the case that John knows that 2 + 2 = 5.'

Presuppositions: Test #2 (ctd.)

Projection test: conditionals

'If Elvis Presley has stopped emailing me, then I'll be able to move on with my life.'

'If John knows that 2 + 2 = 5, then he'll get a gold sticker.'

Projection test: attitude verbs

'My mum hopes that Elvis Presley has stopped emailing me.'

'Karim believes that John knows that 2 + 2 = 5.'

Presuppositions: Definite descriptions

– The tests give weight to the claim that definite descriptions carry presuppositions and can lead to truth value gaps

Definite descriptions and the tests

'The King of France is bald.'

'Hey, wait a minute: but it isn't true that France has a King!'

'#Hey, wait a minute: but it isn't true that he is bald!'

'It's not the case that the King of France is bald.'

'If the King of France is bald, then so too is the King of England.'

'Jake hopes that the King of France is bald'

(v) Future contingents

Sea Battle

(6) 'There will be a sea battle tomorrow.'

– Claim: (6) is neither true nor false.
– Aristotle's argument, roughly put:
  1. If (6) were true or if it were false, this fact would be determined by the state of the world.
  2. Nothing about the state of the world could determine this. (Because of free will, quantum indeterminism,…)
  3. (6) is neither true nor false.
– Note: Aristotle concedes the truth of

(6') 'Either there will be a sea battle tomorrow or there will not.'

(vi) Borderline cases

Colour Gradients

(7) The central dot is green.

– True? False?
– Some say neither: the dot is neither definitely green, nor definitely not.
– This applies to many other 'vague' concepts aside from colour: being tall, being bald, being rich, being happy, etc.

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Some Gappy Logics, Pt 1

Introduction

- We've seen some reasons to countenance truth-value gaps
- How do we suitably generalise our logic to make room for these cases?
- If we go trivalent, we must expand our understanding of
  - (a) how complex sentences get their truth values
  - (b) what it means for an argument to be valid
- Regarding (b), we'll need to decide on a set D of designated truth values and then offer:

  An argument is valid iff there is no possible valuation that assigns to all premises a designated value but assigns to the conclusion a non-designated value.

Introduction (ctd)

- We've seen a partial trivalent t. table for $\supset$
- We'll now look at some more comprehensive proposals, focusing mainly on sentential logic
  - (1) The 'weak Kleene', aka 'Bochvar', logic B3 (Kleene 1952)
  - (2) The 'strong Kleene' logic K3 (Kleene 1952)
  - (3) The 'supervaluationist' logic SV (van Fraassen 1966)
- We'll also discuss
  - (4) A few full trivalent tables for $\supset$ and their interaction with different choices for D
- Note: with 3 values, the number of rows for t. tables is 3^n, so we tables in an alternative compact format

The logic B3

- We have $V = \{0, i, 1\}$ and $D = \{1\}$:

  $\Gamma \models \varphi$ iff there exists no valuation v such that v assigns 1 to all sentences in Γ but assigns 0 or # to φ

  $\Gamma \models \varphi$ iff it is not possible for all the premises to be true but the conclusion not true
- Note that, unlike in the bivalent case, here 'Not true' $\not\Leftrightarrow$ 'False'!!

The logic B3 (ctd)

- The truth tables simply extend the bivalent ones:

$\sim$	
1	0
0	1
#	#

$\vee$	1	0	#
1	1	1	#
0	1	0	#
#	#	#	#

&	1	0	#
1	1	0	#
0	0	0	#
#	#	#	#

$\supset$	1	0	#
1	1	0	#
0	1	1	#
#	#	#	#

$\Rightarrow$ If one of the inputs is #, then so too is the output.

Comments on B3

- B3 is well-suited to presupposition failure: 'nonsense breeds nonsense' or 'garbage in, garbage out'.

Porridge/Rivers

(8) 'All centaurs eat porridge and Melbourne is in Australia.'

(9) ' Either the king of France is bald or rivers run uphill.'

- Truth tables for B3: (8) and (9) are neither true nor false.
- Arguably seems intuitive.

Comments on B3 (ctd)

- Not so good for future contingents and borderline cases:

Pirate/Banker

(10) 'I am a Somali pirate and there will be a sea battle tomorrow.'

(11) ' Either James is bald or he is a banker' (assuming James is a borderline bald banker)

- (10) seems false (right??)
- (11) seems true

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