Introduction to Logic

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Logic and the Dynamics of Belief Pt.1

LA TROBE UNIVERSITY

Last week

- Wrapping up 3-valued logics
  - The logics K3 and SV
  - Complete De Finetti-style tables for the conditional
  - Tableaux for K3

FROM STATICS TO DYNAMICS

Logic and belief

- Some sets of beliefs are rationally permissible to hold in principle and others not
- Key notion in determining this: validity
- Indeed, if we believe certain claims, then
  (1) there are certain other claims that we must also believe
  (because they are consequences of the former)
  (2) there are certain other claims that we cannot believe
  (because they are inconsistent with the former, i.e. their negations are consequences of the former)

Logic and belief (ctd)

Permissible vs impermissible beliefs

If I believe that $A\&B$, then

(1) I must also believe: $A, B, A \supset B, (A\&B) \vee C$, etc.

(2) I cannot also believe: $\sim (A\&B), \sim A, \sim B, A \supset\sim B$, etc.

- In determining validity, logic helps determine which sets of beliefs are rationally permissible to hold in principle

More technically…

- In any state of mind Ψ that one can be in, there is a corresponding belief set $[\Psi]$ containing the sentences that we take to be true
- If we are rational, then $[\Psi]$ will obey:

  (1) Closure under consequence: If $[\Psi] \models \varphi$, then $\varphi \in [\Psi]$

  (2) Consistency: It is not the case that both $\varphi \in [\Psi]$ and $\neg\varphi \in [\Psi]$

- We can also express (1) using the consequence operator Cn:

  $\mathrm{Cn}(\Gamma) = \{\varphi \mid \Gamma \models \varphi\}$

  ($\mathrm{Cn}(\Gamma)$ is the set of sentences φ that are entailed by Γ)

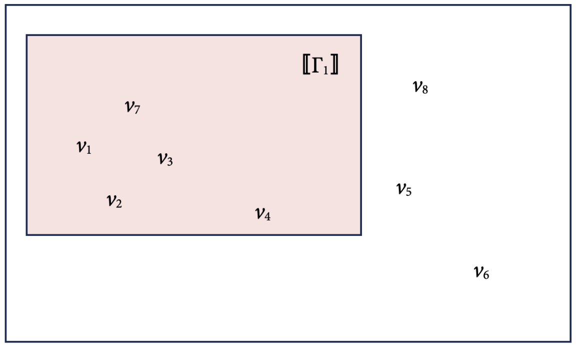
  This gives:

  (1) $\mathrm{Cn}([\Psi]) \subseteq [\Psi]$

Representing sets of sentences

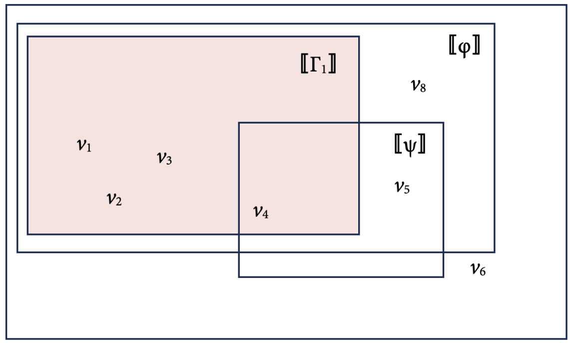
- A set of sentences Γ can be represented by a subset of the set W of all classical valuations (aka models, worlds):

  The set $[\![\Gamma]\!]$ of all valuations that make all sentences in Γ true



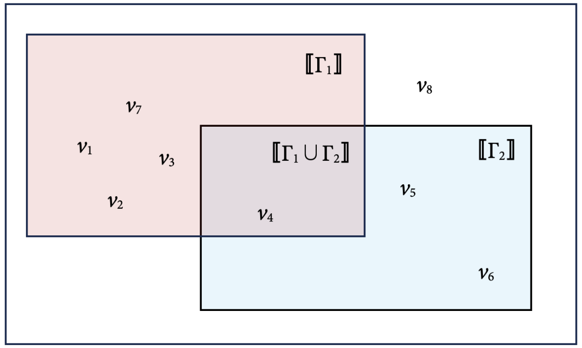
Representing sets of sentences (ctd)

- If Γ is closed under consequence, then a sentence φ is in Γ iff $[\![\Gamma]\!]$ is a subset of its set of models $[\![\varphi]\!]$



Representing sets of sentences (ctd)

- Note: if we add to a set of sentences, we typically subtract from the corresponding set of models!



Aside: Belief sets and SV

- Belief sets can equivalently be equated with a supervaluationist valuation
- Interpretation:

  $v(\varphi) = 1$: φ is believed ($\varphi \in [\Psi]$)

  $v(\varphi) = 0$: φ is disbelieved ($\neg\varphi \in [\Psi]$)

  $v(\varphi) = \#$: φ is neither believed not disbelieved ($\varphi, \neg\varphi \notin [\Psi]$)
- The classical valuations in $[\![[\Psi]]\!]$ are the resolutions of v_a

Going dynamic

- So far: a completely static picture
- But what about dynamics? What about when things change?
- What about when we find out that
  (1) We cannot be committed to a claim A that we previously were committed to?
  (2) We must be committed a claims B that we previously weren't committed to?
- What should our posterior belief set look like, compared to our prior belief set?

An example

Swan

Suppose we are in a state of mind such that we believe:

The swan caught in the trap is European (A)

If the swan is European, then it's white all over ($A \supset B$)

and hence, by deductive inference:

The swan is white all over (B)

An example (ctd.)

Swan (ctd.)

But we then change our views, so that either:

(a) We come to acquire the belief that the swan *isn't* white all over (we revise our beliefs by $\sim B$), or

(b) We come to simply retract the belief that the swan is white all over and suspend judgment (we contract our belief by B)

In either case, we can't hold on to *both* A and $A \supset B$

What should our posterior beliefs look like?

- Logic alone can't answer this question

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The AGM postulates

Belief Revision Theory

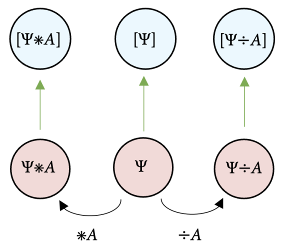
- Belief Revision Theory is a field devoted to modelling the dynamics of rational belief
- It is multidisciplinary venture with independent origins in database management and philosophy
- Canonical model introduced in

  Alchourrón, Gärdenfors & Makinson's 'On the Logic of Theory Change"

- Substantial literature, mainly in computer science

Revision and contraction functions

- In Belief Revision Theory, we define:
  - A revision function $*$ returns the state $\Psi * A$ resulting from adjusting Ψ to accommodate the consistent inclusion of A
  - A contraction function $\div$ returns the state $\Psi \div A$ resulting from adjusting Ψ to accommodate the retraction of A



The AGM postulates

- The AGM postulates of Alchourrón, Gärdenfors and Makinson:

  Rules concerning the relation between prior and posterior belief sets

- They encode an idea of minimal change

The AGM postulates for revision

- The AGM postulates for revision (AGM*):

(K1*) $\mathrm{Cn}([\Psi * A]) \subseteq [\Psi * A]$

(K2*) $A \in [\Psi * A]$

(K3*) $[\Psi * A] \subseteq \mathrm{Cn}([\Psi] \cup \{A\})$

(K4*) If $B \in [\Psi]$, but $B \notin [\Psi * A]$, then $\sim A \in [\Psi]$

(K5*) If A is consistent, then so too is $[\Psi * A]$

(K6*) If $A \Leftrightarrow B$, then $[\Psi * A] = [\Psi * B]$

(K7*) $[\Psi * (A \& B)] \subseteq \mathrm{Cn}([\Psi * A] \cup \{B\})$

(K8*) If $\sim B \notin [\Psi * A]$, then $\mathrm{Cn}([\Psi * A] \cup \{B\}) \subseteq [\Psi * (A \& B)]$

The AGM postulates for revision (ctd)

(K1*) $\mathrm{Cn}([\Psi * A]) \subseteq [\Psi * A]$

Any consequence of a posterior belief set is in that belief set: posterior belief sets are closed under consequence

(K2*) $A \in [\Psi * A]$

After revising by a sentence, that sentence is in the belief set

(K3*) $[\Psi * A] \subseteq \mathrm{Cn}([\Psi] \cup \{A\})$

When revising by a sentence A, we don't add any sentences that go beyond the joint consequences of A and the prior beliefs

The AGM postulates for revision (ctd)

(K4*) If $B \in [\Psi]$, but $B \notin [\Psi * A]$, then $\sim A \in [\Psi]$

If revising by A would remove a belief one holds, then one must initially believe that A is false.

(K4*) can also be expressed as:

If $\sim A \notin [\Psi]$, then $\mathrm{Cn}([\Psi] \cup \{A\}) \subseteq [\Psi * A]$

Together with (K3*), this says:

If the input to revision is consistent with the prior belief set, then the posterior belief set is simply obtained by adding the input to the prior beliefs and taking the logical closure

The AGM postulates for revision (ctd)

(K5*) If A is consistent, then so too is $[\Psi * A]$

Posterior belief sets are consistent (so long as the input to revision is)

(K6*) If $A \Leftrightarrow B$, then $[\Psi * A] = [\Psi * B]$

Revision isn't sensitive to the way an input is presented, syntactically

(K7*) $[\Psi * (A \& B)] \subseteq \mathrm{Cn}([\Psi * A] \cup \{B\})$

This strengthens (K3*) (let B be a tautology)

(K8*) If $\sim B \notin [\Psi * A]$, then $\mathrm{Cn}([\Psi * A] \cup \{B\}) \subseteq [\Psi * (A \& B)]$

This strengthens (K4*) (let B be a tautology)

The AGM postulates for contraction

- The AGM postulates for contraction ($\mathrm{AGM}^{\div}$):

(K1$^{\div}$) $\mathrm{Cn}([\Psi \div A]) \subseteq [\Psi \div A]$

(K2$^{\div}$) $[\Psi \div A] \subseteq [\Psi]$

(K3$^{\div}$) If $A \notin [\Psi]$, then $[\Psi \div A] = [\Psi]$

(K4$^{\div}$) If A isn't a tautology, then $A \notin [\Psi \div A]$

(K5$^{\div}$) If $A \in [\Psi]$, then $[\Psi] \subseteq \mathrm{Cn}([\Psi \div A] \cup \{A\})$

(K6$^{\div}$) If $A \Leftrightarrow B$, then $[\Psi \div A] = [\Psi \div B]$

(K7$^{\div}$) $[\Psi \div A] \cap [\Psi \div B] \subseteq [\Psi \div A \& B]$

(K8$^{\div}$) If $A \notin [\Psi \div A \& B]$, then $[\Psi \div A \& B] \subseteq [\Psi \div A]$

The AGM postulates for contraction (ctd)

(K1$^{\div}$) $\mathrm{Cn}([\Psi \div A]) \subseteq [\Psi \div A]$

Any consequence of a posterior belief set is in that belief set: posterior belief sets are closed under consequence

(K2$^{\div}$) $[\Psi \div A] \subseteq [\Psi]$

Contractions can't lead you to gain new beliefs

(K3$^{\div}$) If $A \notin [\Psi]$, then $[\Psi \div A] = [\Psi]$

If it's already the case that you don't believe A, then contracting by A makes no difference

(K4$^{\div}$) If A isn't a tautology, then $A \notin [\Psi \div A]$

If you contract by A, then you no longer believe A (unless A is a tautology)

The AGM postulates for contraction (ctd)

(K5$^{\div}$) If $A \in [\Psi]$, then $[\Psi] \subseteq \mathrm{Cn}([\Psi \div A] \cup \{A\})$

If you remove a belief and then add it back, you will recover all your original beliefs (aka 'Recovery')

(K6$^{\div}$) If $A \Leftrightarrow B$, then $[\Psi \div A] = [\Psi \div B]$

Contraction isn't sensitive to the way an input is presented, syntactically

(K7$^{\div}$) $[\Psi \div A] \cap [\Psi \div B] \subseteq [\Psi \div A\&B]$

If a belief can survive both contraction by A and contraction by B, it can survive contraction by $A\&B$

The AGM postulates for contraction (ctd)

(K8$^{\div}$) If $A \notin [\Psi \div A\&B]$, then $[\Psi \div A\&B] \subseteq [\Psi \div A]$

If you contract by $A\&B$, you will have to lose either A, or B, or both. So, if you already give up A when you contract by $A\&B$, then you must lose at least as many beliefs when you contract by $A\&B$ as when you simplyy contract by A

The Harper & Levi Identities

- Revision and contraction seem somehow related
- The Harper and Levi Identities connect the two:

(LI) $[\Psi * A] = \mathrm{Cn}([\Psi \div \sim A] \cup \{A\})$

(HI) $[\Psi \div A] = [\Psi] \cap [\Psi * \sim A]$

- Motivation for (LI)
  - Simplest modification of $[\Psi]$ that includes A: $\mathrm{Cn}([\Psi] \cup \{A\})$
  - But $[\Psi]$ and A needn't be consistent: replace $[\Psi]$ by $[\Psi \div \sim A]$
- Informally, (HI) says:
  - Retract anything no longer endorsed, had one come to believe that $\sim A$ (since $\sim A$ considered possible)
  - Retract nothing further and introduce nothing (minimal change)

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AGM and Orderings

TPOs

- These postulates look complex but have a very simple representation in terms of orderings of valuations
- A total preorder (TPO) $\preccurlyeq$ is a binary relation that is

  complete ($x \preccurlyeq y$ or $y \preccurlyeq x$)

  transitive (if $x \preccurlyeq y$ and $y \preccurlyeq z$, then $x \preccurlyeq z$)

TPOs

The relation “lesser than or equal to” $\leq$ on the set of positve integers $\{1, 2, \ldots\}$ is a TPO

TPOs

- The minimal subset of a set W ordered by $\preccurlyeq$ is the set of elements x of W such that for all y in W, $x \preccurlyeq y$

  $\min(\preccurlyeq, W) = \{x \in W \mid \text{if } y \in W, \text{then } x \preccurlyeq y\}$

Minimal subsets

The minimal subset of the set of positive integers ordered by $\leq$ is $\{1\}$

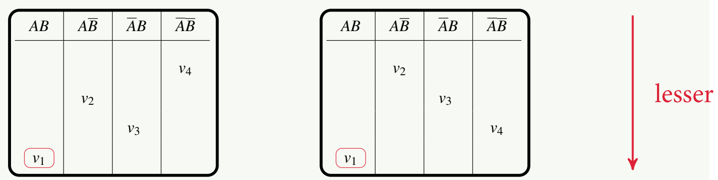
- Every state Ψ is associated with a TPO $\preccurlyeq_\Psi$ over W
- This TPO is often thought of as an ordering of complete possibilities by comparative implausibility ($x \preccurlyeq y = x$ is less implausible or far-fetched than y)

Prior beliefs with orderings

- The set of models of $[\Psi]$ is the set of minimal models

(1) $[\![[\Psi]]\!] = \min(\preccurlyeq_\Psi, W)$

Minimal subsets in *Swan*



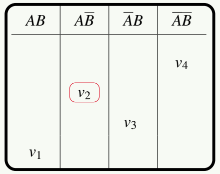
$[\Psi] = \mathrm{Cn}(A \& B)$

Posterior beliefs with orderings: Revision

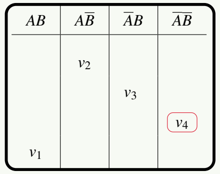
- The set of models of $[\Psi * A]$ is the set of minimal models of A

(2) $[\![[\Psi * A]]\!] = \min(\preccurlyeq_\Psi, [\![A]\!])$

Revision in *Swan*



Retain A but lose $A \supset B$
$[\Psi] = \mathrm{Cn}(A \& \sim B)$



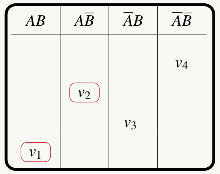
Lose A but retain $A \supset B$
$[\Psi] = \mathrm{Cn}(\sim A \& \sim B)$

Posterior beliefs with orderings: Contraction

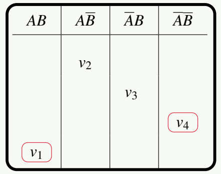
- The set of models of $[\Psi \div A]$ is the union of the set of minimal models with the set of minimal models of $\sim A$

(3) $[\![[\Psi \div A]]\!] = \min(\preccurlyeq_\Psi, W) \cup \min(\preccurlyeq_\Psi, [\![\sim A]\!])$

Contraction in *Swan*



Retain A but lose $A \supset B$
$[\Psi] = \mathrm{Cn}(A)$



Lose A but retain $A \supset B$
$[\Psi] = \mathrm{Cn}(A \equiv B)$