Introduction to Logic

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Logic and the Dynamics of Belief Pt.2

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Last week

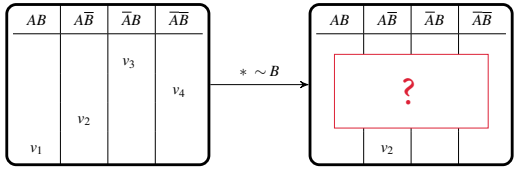
- The role of logic in regulating the beliefs one ought to have or ought to lack at a single point in time (belief sets)
- How to represent belief sets in terms of sets of models
- The need for a framework to handle how belief evolve over time
- The AGM postulates for revision and contraction
- The Harper and Levi identities connecting revision and contraction
- How to represent AGM belief change in terms of orderings of models

Beyond AGM: Iterated Change

AGM and iterated change

- What about beliefs resulting from 2 or more changes, e.g. $[((\Psi * A) \div B) * C]$?
- AGM hardly constrains the result of even 2 changes:

  Only the posterior minimal models are determined, but not the rest of the posterior TPO



- Beyond that, does rationality really place no constraints?

Beyond AGM

- Most people think AGM needs supplementing
- The Darwiche-Pearl (DP) postulates constrain sequences of revisions
- The postulates of Chopra *et al*: similar set but relating to sequences of contractions
- We'll just focus on DP here

The DP postulates: TPOs

- Notation: $x \prec y = x$ is strictly lesser than y ($x \preccurlyeq y$ but not $y \preccurlyeq z$)
- In terms of TPOs:

($C1^*_{\preccurlyeq}$) If $x, y \in [\![A]\!]$ then $x \preccurlyeq_{\Psi * A} y$ iff $x \preccurlyeq_{\Psi} y$

($C2^*_{\preccurlyeq}$) If $x, y \in [\![\sim A]\!]$ then $x \preccurlyeq_{\Psi * A} y$ iff $x \preccurlyeq_{\Psi} y$

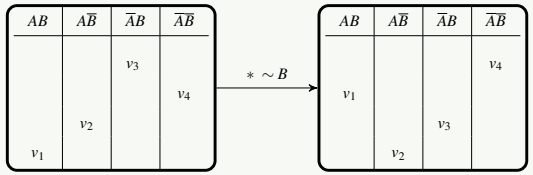
($C3^*_{\preccurlyeq}$) If $x \in [\![A]\!]$, $y \in [\![\sim A]\!]$ and $x \prec_{\Psi} y$, then $x \prec_{\Psi * A} y$

($C4^*_{\preccurlyeq}$) If $x \in [\![A]\!]$, $y \in [\![\sim A]\!]$ and $x \preccurlyeq_{\Psi} y$, then $x \preccurlyeq_{\Psi * A} y$

(Moving from $\preccurlyeq_{\Psi}$ to $\preccurlyeq_{\Psi * A}$ can only involve a 'promotion' of models in $[\![A]\!]$ in relation to models in $[\![\sim A]\!]$)

The DP postulates: TPOs (ctd)

Violations of DP postulates (TPO version)



Problem #1: v_3 has overtaken v_4, violating ($C3^*_{\preccurlyeq}$) and ($C4^*_{\preccurlyeq}$)

Problem #2: v_3 has overtaken v_1, violating ($C2^*_{\preccurlyeq}$)

- OK, but do the DP postulates make sense in terms of belief sets?

The DP postulates: Belief sets

- In terms of belief sets:

($C1^*$) If $A \in \mathrm{Cn}(B)$ then $[(\Psi * A) * B] = [\Psi * B]$

($C2^*$) If $\sim A \in \mathrm{Cn}(B)$ then $[(\Psi * A) * B] = [\Psi * B]$

(Adding an intermediate revision by a weaker or contradictory sentence is redundant)

($C3^*$) If $A \in [\Psi * B]$ then $A \in [(\Psi * A) * B]$

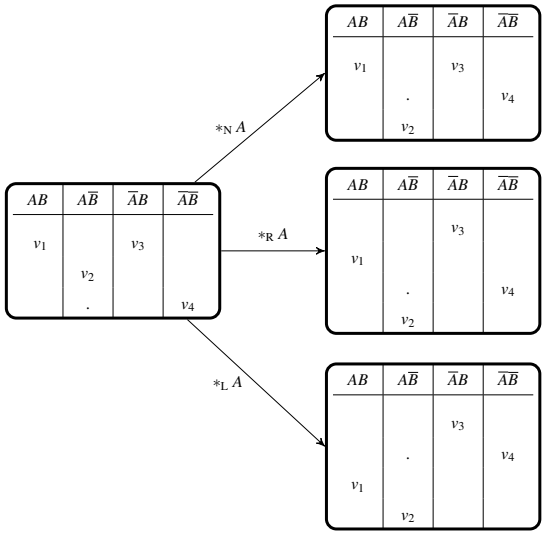
($C4^*$) If $\sim A \notin [\Psi * B]$ then $\sim A \notin [(\Psi * A) * B]$

(Adding an intermediate revision by A doesn't worsen the status of A or improve the status of $\sim A$)

Adding further principles?

- The DP/Chopra *et al* postulates are not strong enough for the prior TPO to fully determine the posterior TPO
- But some have offered further principles such that this determination holds
- This is the case in the 3 popular 'elementary' revision methods:
  - lexicographic ($*_L$) (due to Nayak)
  - restrained ($*_R$) (due to Booth & Meyer)
  - natural ($*_N$) (due to Boutillier)
- The suggested determination of posterior TPO by prior TPO has been criticised (Chandler & Booth)
- Weaker sets of further principles have been proposed instead (Chandler & Booth)

Elementary operators: illustration



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Criticisms of the postulates

Overview

- The postulates we have seen (AGM, DP, Chopra *et al*) are popular, but have been criticised
- Main complaints concern:
  - $(K5^{\div})$, aka "Recovery" (by Hansson and others)
  - $(C2^{*})$ (by Stalnaker and others)
- $(K7^{*})$, $(K7^{\div})$, $(K8^{*})$ and $(K8^{\div})$ have also been criticised (by Rott), but the criticism is hard to present succinctly
- The same applies to $(K4^{*})$, aka "Preservation" (criticised by Rabinowicz)

Criticisms of $(K5^{\div})$

- Recall:

  $(K5^{\div})$ If $A \in [\Psi]$, then $[\Psi] \subseteq \mathrm{Cn}([\Psi \div A] \cup \{A\})$

> *George*
>
> "I previously entertained the two beliefs 'George is a criminal' (C) and 'George is a mass murderer' (M). When I received information that induced me to give up the first of these beliefs (C), the second (M) had to go as well (since C would otherwise follow from B). . . I then received new information that made me accept the belief 'George is a shoplifter' (S)."

- Hansson claims: $M \notin [((\Psi \div C) * S]$
- But $(K5^{\div})$ (+ other principles) entails: $M \in [((\Psi \div C) * S]$

Criticisms of $(K5^{\div})$ (ctd)

- Proof:

(1)	$M \in [\Psi]$	[Assumption]
(2)	$M \in \mathrm{Cn}([\Psi \div C] \cup \{C\})$	[(1), $(K5^{\div})$]
(3)	$C \supset M \in [\Psi \div C]$	[(2), properties of Cn]
(4)	$\sim S \notin [\Psi]$	[Assumption]
(5)	$\sim S \notin [\Psi \div C]$	[(4), $(K2^{\div})$]
(6)	$C \supset M \in [((\Psi \div C) * S]$	[(3), (5) $(K4^{*})$]
(7)	$S \in [((\Psi \div C) * S]$	[$(K2^{*})$]
(8)	$C \in \mathrm{Cn}(S)$	[Assumption]
(9)	$C \in [((\Psi \div C) * S]$	[(7), (8)]
(10)	$M \in [((\Psi \div C) * S]$	[(6), (9)]

Criticisms of $(K5^{\div})$ (ctd)

- But is $[((\Psi \div C) * S]$ the relevant belief set?
- Glaister:

> The situation Hansson has in mind shouldn't be modelled by a simple contraction by C but by a more substantial contraction, e.g.:
>
> $[((\Psi \div C \vee (C \supset M)) * S]$
>
> Since $C \supset M \notin [\Psi \div C \vee (C \supset M)]$, the reasoning will be blocked

Criticisms of (C2*)

- Recall:

  (C2*) If $\sim A \in \mathrm{Cn}(B)$ then $[(\Psi * A) * B] = [\Psi * B]$

  Coins

  Two biased coins are flipped and placed in a box. Based on their bias, I initially believe both landed heads ($H_1 \wedge H_2$). Two reliable witnesses then simultaneously provide independent reports: Carla that coin 1 is not heads up ($\sim H_1$) and Dora that coin 2 is not heads up ($\sim H_2$). Subsequently, Elmer, who is the most reliable overall, reports that coin 1 is heads up (H_1).

- Stalnaker claims: $\sim H_2 \in [(\Psi * \sim H_1 \wedge \sim H_2) * H_1]$
- But (C2*) jointly entails: $\sim H_2 \notin [(\Psi * \sim H_1 \wedge \sim H_2) * H_1]$

Criticisms of (C2*) (ctd)

- Proof:

(1)	$H_1 \& H_2 \in [\Psi]$	[Assumption]
(2)	$H_2 \in [\Psi]$ and $\sim H_1 \notin [\Psi]$	[(1)]
(3)	$H_2 \in [\Psi * H_1]$	[(2), (K4*)]
(4)	$\sim (\sim H_1 \wedge \sim H_2) \in \mathrm{Cn}(H_1)$	[Sentential logic]
(5)	$[\Psi * H_1] = [(\Psi * \sim H_1 \wedge \sim H_2) * H_1]$	[(4), (C2*)]
(6)	$H_2 \in [(\Psi * \sim H_1 \wedge \sim H_2) * H_1]$	[(3), (5)]

Criticisms of (C2*) (ctd)

- But is $[(\Psi * \sim H_1 \wedge \sim H_2) * H_1]$ the relevant belief set?
- Chopra *et al*:

  The situation shouldn't be modelled by revisions by results, but by revisions by reports of results:

  $[(\Psi * C\_says\_not\_H_1 \wedge D\_says\_not\_H_2)) * E\_says\_H_1)]$

  Since the three sentences are atomic, we can't apply $(C2^*_{\preccurlyeq})$

- Delgrande & Jin:

  The first revision shouldn't be modelled by a revision by a single sentence but by a "parallel" revision by a set of sentences:

  $[(\Psi \circledast \{\sim H_1, \sim H_2\}) * H_1]$

  Parallel revision doesn't obey an analogue of $(C2^*_{\preccurlyeq})$ that allows us to claim $[(\Psi \circledast \{\sim H_1, \sim H_2\}) * H_1] = [(\Psi * H_1]$

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