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[12] Indifference (ctd.) + Updating Belief

0. Outline

1. **Responding to the paradoxes of indifference**
2. **Updating belief**

1. Responding to the paradoxes of indifference

- So should we give up on **PI**?
- It would be a shame: it has some prima facie plausibility (at least in some cases: games of chance, etc.).
- Jeffreys [1939] and Jaynes [1968] have suggested that the urn, cube and water/wine paradoxes can be solved by requiring that, in cases of ignorance, the pdf be invariant under various relevant tranformations of the relevant r.v. (e.g. raising it to some power).
- van Fraassen [1989:314] shows that this approach isn't entirely satisfactory.
- Another option: ditch (some of) PROB.
- The paradoxes associated with **PI** all hinge on requiring additivity for d.o.b.s

1. Responding to the paradoxes of indifference

- E.g.: in the urn case, finite additivity precludes having d.o.b of 0 in *W*, *C*, *R* and *B*.
- But if we *could* have that, then **PI** would be satisfied.
  - No epistemic reason to favour any member of $P = \{W, C\}$? No problem: $\text{Bel}_S(W) = \text{Bel}_S(C) = 0$
  - No epistemic reason to favour any member of $P = \{W, R, B\}$? No problem: $\text{Bel}_S(W) = \text{Bel}_S(R) = \text{Bel}_S(B) = 0$
- There is in fact a view that finite additivity is too strong a requirement on graded belief.
- *Dempster-Shafer theory* (DST), a rival of Bayesianism, drops additivity for the weaker requirement of *super-additivity*.
- DST hasn't (yet) been all that widely discussed in philosophy.

1. Responding to the paradoxes of indifference

- On this view, the synchronic requirements on d.o.b.s are:

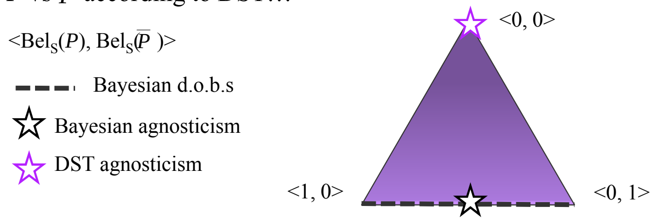
  [DS1]: $\text{Bel}_S(P) \geq 0$

  [DS2]: $\text{Bel}_S(\Omega) = 1$

  [DS3]: If $P \cap Q = \emptyset$ then $\text{Bel}_S(P \cup Q) \geq \text{Bel}_S(P) + \text{Bel}_S(Q)$
- Note that:
  - DST and Bayesianism have *very* different takes on what having a d.o.b. in *P* of 0 involves:
    - DST: not having any confidence that *P* is the case (≈ not believing that *P*).
    - Bayesianism: being certain that *P* isn't the case (≈ believing that not *P*).
  - If Bel_S is a probability function, it is also a DS function.

1. Responding to the paradoxes of indifference

- The following diagram represents the space of possible attitudes to *P* vs $\overline{P}$ according to DST…



- What about justifying the framework?
- How about DBAs etc.?
- There is comparatively little work on the philosophical justification for the DS framework.

1. Responding to the paradoxes of indifference

- Smets [1997], for example, attempts to provide an 'axiomatic justification' for DST (i.e. tries to motivate a set of desiderata *D* for rational belief functions and shows that *Bel* satisfies *D* iff *Bel* is a DS function).
- Wrt DBAs, proponents of DST deny that degrees of belief and valuations/betting dispositions are connected in the way that Bayesians suggest.
- They suggest that fair betting odds *are* determined by d.o.b.s but only via a transformation of these into 'betting probabilities'.
- For more on sets of synchronic constraints on degrees of belief that don't include additivity, see Haenni [ms] (I have a pdf copy if you want one).

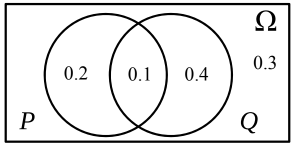
2. Updating belief

- So far: arguments to the effect that our d.o.b.s should obey various synchronic constraints (e.g. **PROB**, **PI**).
- But should our d.o.b.s also obey various *diachronic* constraints? Are there rules for rationally updating one's beliefs over time?
- Presumably, we can at least say the following:

  Certainty: If *S* is rational and, between *t* and *t*+1, learns that *Q* and only that *Q*, then $\text{Bel}_{S,t+1}(Q) = 1$.
- Now if *S* didn't already have this particular d.o.b. at *t*, then given that $\text{Bel}_{S,t+1}$ is required to be a probability function, the required change will in turn have to precipitate further modifications.
- But there are *many* probabilistically coherent belief functions that assign a d.o.b. of 1 in *Q* , i.e. candidates for being $\text{Bel}_{S,t+1}$.

2. Updating belief

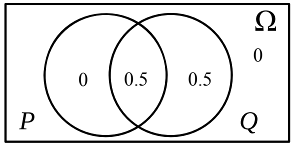
- E.g:
  - At *t*, *S* has the following belief function, defined over the field formed by taking the closure of 'Priscilla hates me' (*P*) and 'Quentin is a liar' (*Q*) under union and negation:
    - $\text{Bel}_{S,t}$



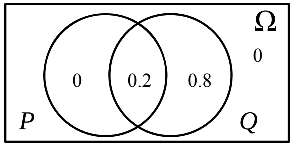
  - At *t*+1, upon learning that Quentin is indeed a liar, *S* must have $\text{Bel}_{S,t+1}(Q) = 1$.

2. Updating belief

- But that leaves open a choice between, say:
  - $\text{Bel}^1_{S,t+1}$



  - $\text{Bel}^2_{S,t+1}$



- Is *S* rationally required to pick any particular one of these, or will any one of them do?

2. Updating belief

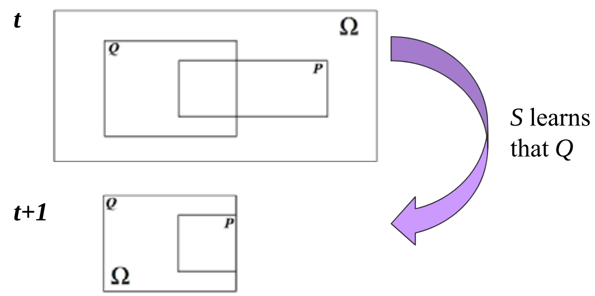
- Many Bayesians think that there *is* a particular choice prescribed:

  SC (strict conditionalisation): If *S* is rational and, between times *t* and *t*+1 learns that *Q* and only that *Q*, then for all $P \in \mathcal{F}$, $\text{Bel}_{S,t+1}(P) = \text{Bel}_{S,t}(P|Q)$.

  (provided $\text{Bel}_{S,t}(P|Q)$ is well-defined; not an issue if we buy into one of the non-Kolmogorovian axiomatisations mentioned in L4/L5)
- In other words: upon learning that *P* and only that *P*, you ought to set your new unconditional d.o.b.s to the values of your previous d.o.b.s conditional on *Q*.
- Assuming that $\text{Bel}_{S,t}$ is a probability function, this particular updating rule satisfies our requirement that upon learning that *Q*, S's d.o.b. in *Q* should move to 1. (as $\text{Bel}_{S,t}(Q|Q) = 1$)

2. Updating belief

- In terms of our previous example, this rule uniquely picks out $\text{Bel}^2_{S,t+1}$. Try to verify this at home…
- **SC** is easiest to visualise by using a Venn diagram in which the probabilities are proportional to the areas:



2. Updating belief

- Some things to note about **SC**. It entails:
  - **Commutativity**: if *S* updates her beliefs in this way, the order of learning doesn't matter, i.e. *S*'s final belief function is the same whether

    (i) *S* learns that *Q* between *t* and *t*+1 and learns that *R* between *t*+1 and *t*+2,

    or

    (ii) *S* learns that *R* between *t* and *t*+1 and learns that *Q* between *t*+1 and *t*+2.
  - **Rigidity**: If *S* is rational and, between *t* and *t*+1, learns that *Q* and only that *Q*, then, for all $P \in \mathcal{F}$, $\text{Bel}_{S,t+1}(P|Q) = \text{Bel}_{S,t}(P|Q)$.

2. Updating belief

- Note: in fact, this particular update procedure is logically equivalent to the conjunction of **Certainty** and **Rigidity**.
- After a number of years during which the rule was taken as too obvious to require defending (!), there are now a number of arguments offered in favour of it.
- The best known one, surprise surprise… the diachronic DBA.
- Essentially the same idea as the previous ones.
- I'll spare you the proof. There is a straightforward sketch in the Howson & Urbach reading on the moodle.
- The diachronic DBA is however more contentious that its synchronic counterpart.

2. Updating belief

- For discussion and some further references see Hajek [forth.: section 5].
- **SC** makes recommendations regarding what to do when we come to *know* that *Q* and thereby should be *subjectively certain* that *Q*.
- What of changes to one's epistemic situation that don't mandate such a high degree of confidence?
- Clearly: such changes sometimes occur.
- Arguably: such changes are the norm.
- Some critics of 'classical foundationalism' even claim that we aren't entitled to be absolutely certain that our phenomenal beliefs are true (e.g. believing that one is in pain, etc.).

2. Updating belief

- E.g. (from Jeffrey [1965]):
  - At *t*, I am wondering about the colour of some piece of cloth in view of making some curtains.
  - I have $\text{Bel}_t(\textit{Green}) = \text{Bel}_t(\textit{Blue}) = 0.3$ and $\text{Bel}_t(\textit{Violet}) = 0.4$.
  - I also have $\text{Bel}_t(\textit{Suit}|\textit{Green}) = 1$, $\text{Bel}_t(\textit{Suit}|\textit{Blue}) = 0.2$, $\text{Bel}_t(\textit{Suit}|\textit{Violet}) = 0$ (where *Suit* = would suit my living room).
  - Between *t* and *t*+1 I take a look at the cloth, but I do so in non-ideal perceptual conditions (under candlelight, say).
  - But now it seems that my experience doesn't justify certainty in any of the propositions; it merely warrants $\text{Bel}_{t+1}(\textit{Green}) = 0.7$, $\text{Bel}_{t+1}(\textit{Blue}) = 0.25$ and $\text{Bel}_{t+1}(\textit{Violet}) = 0.05$.
  - What should my credence in *Suit* be at *t*+1?

2. Updating belief

- Jeffrey [1965] suggests that we generalise **SC**:

  JC (Jeffrey conditionalisation): If S is rational and, between times t and t+1, S's epistemic situation changes so as to rationally require S to have $\text{Bel}_{S,t+1}(Q_1)$, $\text{Bel}_{S,t+1}(Q_2)$,…, $\text{Bel}_{S,t+1}(Q_n)$, where $\{Q_1, Q_2,\ldots,Q_n\}$ is a partition of Ω, then for all $P \in \mathcal{F}$, $\text{Bel}_{S,t+1}(P) = \Sigma_i \text{Bel}_{S,t}(P|Q_i) \times \text{Bel}_{S,t+1}(Q_i)$.
- Wrt our example, we get:

$$\text{Bel}_{t+1}(Suit) = \text{Bel}_{t+1}(Green) \times \text{Bel}_t(Suit|Green) + \text{Bel}_{t+1}(Blue) \times \text{Bel}_t(Suit|Blue) + \text{Bel}_{t+1}(Violet) \times \text{Bel}_t(Suit|Violet)$$

$$= 0.7 \times 1 + 0.25 \times 0.2 + 0 \times 0.05 = 0.75$$

2. Updating belief

- **SC** is simply the special case in which the change mandated by S's new epistemic circumstances involves:
  - the partition $\{Q, \overline{Q}\}$
  - the change to $\text{Bel}_{S,t+1}(Q) = 1$ and $\text{Bel}_{S,t+1}(\overline{Q}) = 0$.

  We have, for all $P \in \mathcal{F}$,

$$\text{Bel}_{S,t+1}(P) = \text{Bel}_{S,t}(P|Q) \times \text{Bel}_{S,t+1}(Q) + \text{Bel}_{S,t}(P|\overline{Q}) \times \text{Bel}_{S,t+1}(\overline{Q})$$

$$= \text{Bel}_{S,t}(P|Q) \times 1 + \text{Bel}_{S,t}(P|\overline{Q}) \times 0$$

$$= \text{Bel}_{S,t}(P|Q)$$

2. Updating belief

- This more general rule has also been given a DBA, which is also extremely controversial.

Reference

- Haenni, R. [ms.] 'Non-additive degrees of belief'
- Hajek, A. [forthcoming]: 'Dutch Book Arguments', in P. Anand, P. Pattanaik, and C. Puppe (eds.) *The Oxford Handbook of Corporate Social Responsibility*.
- Howson, C. & P. Urbach [1993]: *Scientific Reasoning: the Bayesian approach, 2nd Edition*. LaSalle: Open Court.
- Jeffrey, R. [1965]: *The Logic of Decision*. Chicago: University of Chicago Press.
- Smets, P. [1997]: 'The Normative Representation of Quantified Beliefs by Belief Functions'. *Artificial Intelligence* 92: 229-242

Next lecture: 'Confirmation'

- Reading:
  - Fitelson, B. [2000] *Studies in Bayesian Confirmation Theory*, PhD thesis, University of Wisconsin Madison. pp4-8. (the basics of Bayesian confirmation theory)
- Supplementary reading:
  - Earman, J. & W. Salmon[1999] 'The Confirmation of Scientific Hypotheses', in M. Salmon et al. (eds.) *Introduction to the Philosophy of Science*, Indianapolis: Hackett Publishing Company. Sections 2.2 – 2.4. (the basics of pre-Bayesian confirmation theories, which the Bayesian approach supposedly improves on)