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[19] The Lottery (ctd.)

0. Outline

1. Graded vs Full belief – some options (ctd.)
2. The Lottery Paradox

1. Graded vs Full belief – some options

- Recap of last time:
  - Two models of belief: full vs partial.
  - Characterisation of a rational full belief function BEL:
    - **[F1]**: $BEL_s(\Omega) = 1$
    - **[F2]**: $BEL_S(\emptyset) = 0$
    - **[F3]**: $BEL_S(P \cup Q) \geq BEL_S(P) + BEL_S(Q) - BEL_S(P \cap Q)$
    - **[F4]**: If $BEL_s(P) = 1$ and $BEL_s(Q) = 1$, then $BEL_s(P \cap Q) = 1$
  - Generic proposal connecting d.o.b.s with full belief:
    - **Locke**: $BEL_S(P) = 1$ iff $Bel_S(P) > t$, for suitably high t.
  - The case against requiring $t = 1$ (‘*Suggestion 1*’)

1. Graded vs Full belief – some options

- *Suggestion 2*: sufficiently high = t or greater, for some $t < 1$.
- *Worry 1*: arbitrariness - which t? More than 0.5? More than 0.9?
- *Worry 2*: explanatory idleness

  ‘One could easily enough define a concept of belief which identified it with high [degree of confidence] ([degree] greater than some specified number between one-half and one), but it is not clear what the point of doing so would be. Once a [degree of confidence] is assigned to a proposition, there is nothing more to be said... Probabilist decision theory gives a complete account of how [degrees of belief], including high ones, ought to guide... So what could be the point of selecting an interval near the top of the probability scale and conferring on the propositions whose probability falls in that interval the honorific title ‘believed’?’ (Stalnaker, quoted in Sturgeon [2008], see also Frankish [forth.:7])

2. The Lottery Paradox

- *Worry 3*: the Lottery Paradox (Kyburg [1961], Hempel [1962]).
- More specifically, the endorsement of **PROB** and **Locke** motivates the endorsement of jointly inconsistent consequences.
- This is how it goes, where we have decided that the suitably high sub-unit threshold value $t = 0.9$ (the argument doesn't hinge on this particular choice):

  (1) **Locke**: *S* believes that *P* iff *S*'s d.o.b. in $P > 0.9$. (prem.)

  (2) **PROB**: if *S* is rational, then Bel_S is a probability function.

  (3) If *S* is rational, then (a) if *S* believes that *P* and *S* believes that *Q*, then *S* believes that *P* and *Q* (i.e. **[F4]**), (b) *S* doesn't believe any contradictions (i.e. **[F2]**). (prem.)

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(4) Assume, for reductio, that some possible person, *S*, is both rational and subjectively certain that there is a fair 1 000-ticket lottery with one guaranteed winner being held. (assumption)

(5) For each ticket in the lottery, *S* has degree of confidence of 0.999 that it will loose. (from (2) and (4))

(6) For each ticket in the lottery, *S* believes that it will lose. (from (1) and (5)).

(7) *S* believes that every ticket will lose. (from (3)(a) and (6))

(8) *S* believes that some ticket will win. (from (1) and (4))

(9) *S* believes the contradiction that every ticket will lose and that some ticket will win. (from (7), (8) and (3)(a))

2. The Lottery Paradox

(10) No person can be rational and subjectively certain that there is a fair 1 000-ticket lottery with one guaranteed winner being held. (by reductio from (1) to (9))

(11) It is possible to be rational and subjectively certain that there is a fair 1 000-ticket lottery with one guaranteed winner being held. (prem)

- (10) and (11) are jointly inconsistent!
- Something has to go… but *what*?
- Everything looked so appealing!
- Note #1: the argument generalises to any $t < 1$. All you need is a lottery with *n* tickets such that $1 - (1/n) > t$.

2. The Lottery Paradox

- Note #2: if one countenances *infinitary* fair lotteries (hence rejecting countable additivity), it turns out that the argument works even if $t = 1$.
- Why? Because we would, for each ticket, have d.o.b. of 1 in the ticket's losing but d.o.b. of 0 in the associated conjunctive proposition.
- Back to out problem: what are we going to drop?
- *Suggestion #1*: get rid of **[F4]**, allowing one to believe that *P* and believe that *Q* without believing that *P&Q* (e.g. Kyburg [1961]).
- Obviously, we then need to re-axiomatise BEL; we can still make use of weaker requirements, such as:

  [F5]: If $P \subseteq Q$, then $BEL_S(P) \leq BEL_S(Q)$. (analogue of [T6])

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- Kaplan [1981:309] points out that this has the ironic consequence of somewhat undermining the force of the paradox.
- It no longer follows that if one endorses

  (10) No person can be rational and subjectively certain that there is a fair 1 000-ticket lottery with one guaranteed winner being held.

  and also endorses

  (11) It is possible to be rational and subjectively certain that there is a fair 1 000-ticket lottery with one guaranteed winner being held.

  that one thereby endorses their conjunction and therefore violates the rational requirement on not believing a contradiction (**[F2]**).

2. The Lottery Paradox

- *Suggestion #2*: Get rid of **Locke**, replacing it with a weaker alternative that includes some kind of relevant proviso:

  Locke*: $\text{BEL}_S(P) = 1$ iff (i) $\text{Bel}_S(P) > t$, where t = suitably high but < 1 and (ii) condition C doesn't hold.
- For instance, Douven [2002] suggests something like:

  Restriction: $C = P$ is a member of a probabilistically self-undermining set for S.

  A probabilistically self-undermining set for $S =_{def}$ a set of propositions such that for each member of that set, (i) S's d.o.b. in that member $> t$, but (ii) S's d.o.b in that member conditional on the conjunction of all remaining members $\leq t$.

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- E.g.: in the context of our lottery example,

  'That ticket 1 will lose'

  is a member of the set of propositions of the form 'ticket *i* will lose' (L_i) (where $1 \leq i \leq 1\,000$), such that

  (i) For each ticket, S's d.o.b. in that ticket losing = 0.999 > 0.9.

  (ii) For each ticket, S's d.o.b. in that ticket losing conditional on the remainder of the tickets losing = 0 ≤ 0.9.
- There have been *many* other proposals of this kind (e.g. Pollock [1995]).
- Problem: the move comes with extremely undesirable consequences (Douven & Williamson [2006:5-6])…

2. The Lottery Paradox

- Surprising fact:

  It *also* follows from the conjunction of **Locke*** and **Restriction** that no proposition P and agent S is such that $t < \text{Bel}_S(P) < 1$ and $\text{BEL}_S(P) = 1$: i.e. no agent fully believes something that they are not 100% certain of !!!!
- To see why, consider (again, assumption that $t = 0.9$ for expository purposes)
  - Some arbitrary proposition P (e.g. Adam is a spy) and agent S, such that $t < \text{Bel}_S(P) < 1$.
  - The set of propositions *SET* corresponding to the intersections of $\overline{P}$ with various propositions of the form 'ticket *i* will loose' (I assume that we are referring to tickets in a fair lottery with 1 000 tickets and a guaranteed winner).

2. The Lottery Paradox

- Now, assuming that Bel_S is a probability function, for any *i*, such that $1 \leq i \leq 1\,000$:

  $Bel_S(\overline{P} \cup$ ticket *i* will lose$) > 0.9$

  Proof: for any *P* and *Q*, $Bel_S(P \cup Q) \geq Bel_S(P)$ and for any *i*, Bel_S(ticket *i* will lose) $= 0.999 > 0.9$. ■
- So, given that, as stipulated earlier, it is also the case that $Bel_S(P) > 0.9$, it follows that:

  P is a member of a set of propositions, namely $SET \cup \{P\}$, such that, for each member of that set, *S* has a d.o.b. > 0.9 in that member.
- In other words: *P* meets clause (i) of **Restriction.**
- Let's now show that it meets clause (ii)!

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- *S*'s d.o.b in *P* conditional on the conjunction of all the propositions of the form $\overline{P} \cup$ ticket *i* (i.e. the conjunction of all the members of *SET*) will lose is equal to $0 \leq t$. Why?
- It is a law of set theory (see L2, slide 6: distributive laws (i)) that $(P \cup Q_1) \cap (P \cup Q_2) \cap (P \cup Q_3) \ \ldots \ = P \cup (Q_1 \cap Q_2 \cap Q_3 \ldots)$
- Hence the conjunction of the members of $SET = \overline{P} \cup$ (ticket 1 will lose $\cap$ ticket 2 will lose $\cap \ldots \cap$ ticket 1 000 will lose) $= \overline{P} \cup \emptyset$ (as some ticket has to win) $= \overline{P}$
- Conditional on *this*, of course, *S*'s d.o.b in *P* is 0.
- Furthermore, for every proposition of the form $\overline{P} \cup$ ticket *i* will lose, *S*'s d.o.b in that proposition, conditional on the conjunction of *P* and the remaining propositions's of the same form is also equal to $0 \leq t$.

2. The Lottery Paradox

- Why?
- The conditional d.o.b in question is equal to (i) the unconditional d.o.b in the conjunction of the members of $SET \cup \{P\}$ divided by (ii) the unconditional d.o.b in the proposition conditionalised upon (remember: $Pr(P|Q) = Pr(P \cap Q) / Pr(Q)$).
- But the former is equal to 0, as, as we have seen, the conjunction of the members of *SET* amounts to the negation of *P*, and, in turn, the conjunction of *this* with *P* is a contradiction.
- So the ratio as a whole is also equal to $0 \leq t$.
- So every member of $SET \cup \{P\}$ (each one of of the members of *SET*, as well as *P*) is such that *S*'s d.o.b in that member conditional on the conjunction of all remaining members $= 0 \leq t$.

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- So *P* meets clause (ii) of **Restriction**.
- So no agent fully believes something that they are not 100% certain of: Douven's proposal boils down to setting the threshold at 1. ■
- Suggestion: can we perhaps weaken *C*, whilst hanging on to $t < 1$, in such a way as to avoid this result?
- Williamson & Douven [2006] give us good general reasons to believe that this isn't possible (warning: their article is quite heavy-going).

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- So how on earth *do* we solve the lottery paradox??!?

Reference

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