

ELEMENTS OF DEDUCTIVE LOGIC

15. Predicate Logic: tableau methods

J. Chandler

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Introduction

- Last time: wrapping up the semantics for \mathcal{L}_P + introducing tableaux
 - Truth in a model: the general case
 - Truth in *finite* models
 - Testing for validity: a special case
 - Testing for validity: the general case (tbc)
- This time: more on the last point, i.e. tableaux

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Universal formulae

- The rule:

- Rationale:
 - If $(\forall x)A$ is true, then all of its instances are true.
 - This includes all those instances involving names already used on the branch.

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$$\begin{array}{c} (\forall x)A \\ | \\ A(x := a) \\ (a \text{ already on branch}) \end{array}$$

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Negated universal formulae

- The rule:
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 - If $\sim (\forall x)A$ is true, then $(\forall x)A$ is false, so not all of the instances of the latter are true, so at least one of them is false and hence its negation is true.
 - This might not be an instance that involves a name already used on the branch. So we introduce a new name, just in case.

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$$\begin{array}{c} \sim (\forall x)A \\ | \\ \sim A(x := a) \\ (a \text{ new}) \end{array}$$

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Particular vs general rules

- The rules for $(\exists x)A$ and $\sim (\forall x)A$ are known as **particular** rules:
They are applied only *once* to a given formula.
- Once used, we put a tick next to the formula alongside the name introduced: e.g. $\checkmark a$
- The rules for $\sim (\exists x)A$ and $(\forall x)A$ are known as **general** rules:
They can be applied *repeatedly* to one same formula
- After first use, we put a backslash next to the formula alongside the relevant name: e.g. $\backslash a$.
- Upon subsequent uses, we simply add the relevant names e.g. $\backslash a, b, c \dots$

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Note on general rules

- Question:

What happens when all the formulae at the root of the tableau are of the form $\sim (\exists x)A$ or $(\forall x)A$ and do not contain any names?

- Answer:

- Since every domain has at least one element and every element is named, we have to have at least one name by default, say ' a '.
- We then substitute this into all the relevant formulae, using the relevant rules.

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Completedness

- Note: there are no repeatable rules in propositional logic.
- The introduction of these rules makes a difference to the definition of a **completed** tree in pred. logic.
- In prop. logic:
 - A tree is completed iff, in every open branch b , every formula on b that could have had a rule applied to it has had a rule applied to it.
- This is not good enough here: some formulae sometimes need to have rules applied to them more than once.
- In pred. logic:
 - A tree is completed iff, in every open branch b , (i) every formula on b that could have had a rule applied to it has had a rule applied to it *and* (ii) every name on b has been substituted into every formula of the form $\sim (\exists x)A$ or $(\forall x)A$.

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Brief general tips

- Apply propositional rules first, starting with non-branching rules.
- Then apply instantiation rules, starting with particular rules.
- These recommendations are *defeasible*, however: e.g. the application of a general rule may immediately lead to tableau closure.
- Now for 3 examples: a tableau that closes, an open tableau with countermodel and a little surprise. . .

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- We show that $(\forall x)(Fx \supset Gx), (\exists x) \sim Gx \vdash (\exists x) \sim Fx$

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$$(\forall x)(Fx \supset Gx)$$

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$$(\forall x)(Fx \supset Gx)$$
$$(\exists x) \sim Gx \quad \checkmark a$$
$$\sim (\exists x) \sim Fx$$
$$\quad |$$
$$\sim Ga$$

A closed tableau

- We show that $(\forall x)(Fx \supset Gx), (\exists x) \sim Gx \vdash (\exists x) \sim Fx$

$$\begin{array}{c}
 (\forall x)(Fx \supset Gx) \setminus a \\
 (\exists x) \sim Gx \checkmark a \\
 \sim (\exists x) \sim Fx \\
 | \\
 \sim Ga \\
 | \\
 Fa \supset Ga
 \end{array}$$

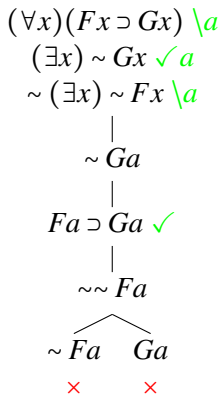
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A completed open tableau

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$(\exists x)Fx$

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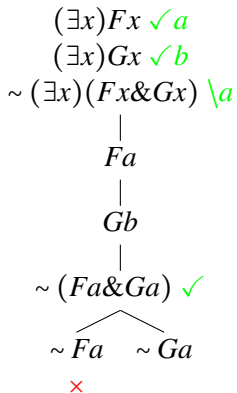
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 (\exists x)Gx \quad \checkmark b \\
 \sim (\exists x)(Fx \& Gx) \quad \backslash a \\
 \quad | \\
 \quad Fa \\
 \quad | \\
 \quad Gb \\
 \quad | \\
 \quad \sim (Fa \& Ga)
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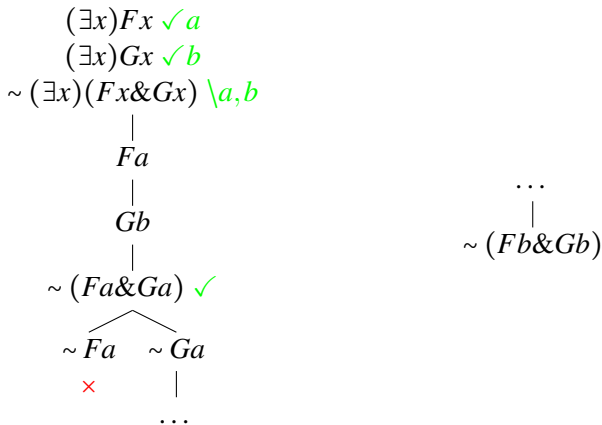
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$(\exists x)Fx$ ✓ *a*
 $(\exists x)Gx$ ✓ *b*
 $\sim (\exists x)(Fx \& Gx)$ \ *a, b*

Fa
 Gb
 $\sim (Fa \& Ga)$ ✓
 $\sim Fa$ ×
 $\sim Ga$
 \dots

\dots
 $\sim (Fb \& Gb)$ ✓
 $\sim Fb$ ↑
 $\sim Gb$ ×

A completed open tableau: the countermodel

- Our open branch contains: Fa , Gb , $\sim Ga$ and $\sim Fb$.
- So we have a domain D of size 2, say $D = \{d, e\}$, with I defined as follows:

I	
a	d
b	e

$I(F)$	
d	1
e	0

$I(G)$	
d	0
e	1

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- So we have a domain D of size 2, say $D = \{d, e\}$, with I defined as follows:

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a	d
b	e

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An interminable open tableau (!) (ctd.)

- What's going on here?
- It turns out that there *exists* a proof of invalidity: the completed tableau is in fact open.
- Problem: the completed open tableau is infinitely long!
- So we *cannot find* this proof in a finite number of steps by sequentially applying rules until we complete the tableau.
- This is also demonstrably true of any other mechanisable procedure for finding a proof of either validity or invalidity in pred. logic.
- We say that predicate logic is **undecidable**.
- This is *not* the case for propositional logic, nor is it the case for the restriction of predicate logic to monadic predicates.
- In both those cases, completed trees are always finite.

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Next session

- Tableau exercises: check Toledo.
- Session after that: finishing off tableaux + identity and definite descriptions.
- Reading: Restall Ch. 11.