ELEMENTS OF DEDUCTIVE LOGIC

5. More on truth tables

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Truth tables for 'complicated' sentences

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- With all these tables in hand, we can now construct truth tables for *any* kind of sentence whatsoever...

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Truth tables for ‘complicated’ sentences: Example 1

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p	q	$((p$	$\equiv$	$q)$	$\&$	$p)$	$\supset$	q

Step 1

Draw up a table: $r = 2^n$ rows, where n is the number of atomic sentences. (Here: $r = 4$.)

Column headers: atomic sentences on the left, complex sentence on the right.

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- The construction of a table for $((p \equiv q)\&p) \supset q$:

p	q	$((p$	$\equiv$	$q)$	$\&$	$p)$	$\supset$	q
1	1							
1	0							
0	1							
0	0							

Step 2

Fill in the columns for the atomic sentences on the left:

- First column: alternate '1' $\times r/2$ and '0' $\times r/2$. (Here: $r/2 = 2$)
- Second column: alternate '1' $\times r/4$ and '0' $\times r/4$. (Here: $r/4 = 1$)
- …

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1	1	1		1				
1	0	1		0				
0	1	0		1				
0	0	0		0				

Step 3

Work your way up the parse tree, filling in the values for increasingly more complex sentences

- For atomic sentences: copy-paste the values under the letter
- For complex sentences: write the values under the main connective, using the relevant. tables

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1	1	1	1	1				
1	0	1	0	0				
0	1	0	0	1				
0	0	0	1	0				

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1	1	1	1	1		1		
1	0	1	0	0		1		
0	1	0	0	1		0		
0	0	0	1	0		0		

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1	0	1	0	0	0	1		
0	1	0	0	1	0	0		
0	0	0	1	0	0	0		

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0	1	0	0	1	0	0		1
0	0	0	1	0	0	0		0

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0	1	0	0	1	0	0	**1**	1
0	0	0	1	0	0	0	**1**	0

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- Let's do another one: $((p \supset q)\&(q \supset r))\&(p\& \sim r)$.

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1	1	1												
1	1	0												
1	0	1												
1	0	0												
0	1	1												
0	1	0												
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1	1	1	1		1									
1	1	0	1		1									
1	0	1	1		0									
1	0	0	1		0									
0	1	1	0		1									
0	1	0	0		1									
0	0	1	0		0									
0	0	0	0		0									

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1	1	1	1	1	1									
1	1	0	1	1	1									
1	0	1	1	0	0									
1	0	0	1	0	0									
0	1	1	0	1	1									
0	1	0	0	1	1									
0	0	1	0	1	0									
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1	1	1	1	1	1		1		1					
1	1	0	1	1	1		1		0					
1	0	1	1	0	0		0		1					
1	0	0	1	0	0		0		0					
0	1	1	0	1	1		1		1					
0	1	0	0	1	1		1		0					
0	0	1	0	1	0		0		1					
0	0	0	0	1	0		0		0					

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1	1	1	1	1	1		1	1	1					
1	1	0	1	1	1		1	0	0					
1	0	1	1	0	0		0	1	1					
1	0	0	1	0	0		0	1	0					
0	1	1	0	1	1		1	1	1					
0	1	0	0	1	1		1	0	0					
0	0	1	0	1	0		0	1	1					
0	0	0	0	1	0		0	1	0					

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1	1	1	1	1	1	1	1	1	1					
1	1	0	1	1	1	0	1	0	0					
1	0	1	1	0	0	0	0	1	1					
1	0	0	1	0	0	0	0	1	0					
0	1	1	0	1	1	1	1	1	1					
0	1	0	0	1	1	0	1	0	0					
0	0	1	0	1	0	1	0	1	1					
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1	1	1	1	1	1	1	1	1	1					1
1	1	0	1	1	1	0	1	0	0					0
1	0	1	1	0	0	0	0	1	1					1
1	0	0	1	0	0	0	0	1	0					0
0	1	1	0	1	1	1	1	1	1					1
0	1	0	0	1	1	0	1	0	0					0
0	0	1	0	1	0	1	0	1	1					1
0	0	0	0	1	0	1	0	1	0					0

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1	1	1	1	1	1	1	1	1	1				0	1
1	1	0	1	1	1	0	1	0	0				1	0
1	0	1	1	0	0	0	0	1	1				0	1
1	0	0	1	0	0	0	0	1	0				1	0
0	1	1	0	1	1	1	1	1	1				0	1
0	1	0	0	1	1	0	1	0	0				1	0
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0	0	0	0	1	0	1	0	1	0				1	0

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1	1	0	1	1	1	0	1	0	0		1		1	0
1	0	1	1	0	0	0	0	1	1		1		0	1
1	0	0	1	0	0	0	0	1	0		1		1	0
0	1	1	0	1	1	1	1	1	1		0		0	1
0	1	0	0	1	1	0	1	0	0		0		1	0
0	0	1	0	1	0	1	0	1	1		0		0	1
0	0	0	0	1	0	1	0	1	0		0		1	0

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1	1	0	1	1	1	0	1	0	0		1	1	1	0
1	0	1	1	0	0	0	0	1	1		1	0	0	1
1	0	0	1	0	0	0	0	1	0		1	1	1	0
0	1	1	0	1	1	1	1	1	1		0	0	0	1
0	1	0	0	1	1	0	1	0	0		0	0	1	0
0	0	1	0	1	0	1	0	1	1		0	0	0	1
0	0	0	0	1	0	1	0	1	0		0	0	1	0

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1	1	0	1	1	1	0	1	0	0	**0**	1	1	1	0
1	0	1	1	0	0	0	0	1	1	**0**	1	0	0	1
1	0	0	1	0	0	0	0	1	0	**0**	1	1	1	0
0	1	1	0	1	1	1	1	1	1	**0**	0	0	0	1
0	1	0	0	1	1	0	1	0	0	**0**	0	0	1	0
0	0	1	0	1	0	1	0	1	1	**0**	0	0	0	1
0	0	0	0	1	0	1	0	1	0	**0**	0	0	1	0

Some important terms

- Each row of the table represents a valuation: an assignment of truth values to the argument's sentences / subsentences that is consistent with the t. tables for the connectives.
- Formally, a valuation v is a function: it maps each sentence / subsentence onto a single truth value.
- If v assigns the value 1 to φ, we write $v(\varphi) = 1$. We then say that v satisfies, or is a model of, φ.
- If v assigns the value 0 to φ, we write $v(\varphi) = 0$, of course.
- Under the main connective of Example 1, we find a column of 1's: the sentence is true on all valuations.
- The sentence is a tautology.

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Some important terms (ctd.)

- Under the main connective of Example 2, we find a column of 0's: the sentence is false on all valuations.
- The sentence is a contradiction.
- If a sentence is neither a tautology nor a contradiction, it is contingent.

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Two more tables…

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p	q	$\sim$	$(p$	$\&$	$\sim$	$q)$
1	1					
1	0					
0	1					
0	0					

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1	0					0
0	1					1
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1	0				1	0
0	1				0	1
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1	0				
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p	q	$\sim$	p	$\vee$	q
1	1	0	1	**1**	1
1	0	0	1	**0**	0
0	1	1	0	**1**	1
0	0	1	0	**1**	0

An interesting observation

- The two previous sentences have the same values under their main connectives.
- We say that they are logically equivalent to each other: $\sim (p \& \sim q) \Leftrightarrow \sim p \vee q$.
- Another way of putting this: $\sim (p \& \sim q) \equiv \sim p \vee q$ is a tautology.
- But they are also logically equivalent to *something else*, right?

p	q	$p \supset q$
1	1	1
1	0	0
0	1	1
0	0	1

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An interesting observation

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- These two equivalences highlight two lines of argument for the truth table for ⊃.
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Truth tables for arguments

- We can apply the method of truth tables to prove the validity or invalidity of an argument form.
- We draw up, side by side, the sub-tables for the various $\mathcal{L}_S$ sentences of the argument form.
- Table for the disjunctive syllogism:

p	q	p	$\vee$	q	$\sim$	p	q
1	1	1	**1**	1	**0**	1	**1**
1	0	1	**1**	0	**0**	1	**0**
0	1	0	**1**	1	**1**	0	**1**
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Truth tables for arguments (ctd.)

- We then consider the question:

  Is there a valuation v such that, for any premise φ, we have $v(\varphi) = 1$, but, for the conclusion ψ, we have $v(\psi) = 0$?

- If NO, then the form is valid.
- Where $\{\varphi_1, \ldots, \varphi_n\}$ is the set of premises and ψ the conclusion, we then write $\{\varphi_1, \ldots, \varphi_n\} \vDash \psi$.
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A valid form

- We can now see that the disjunctive syllogism is valid: $p \vee q, \sim p \vDash q$

p	q	p	$\vee$	q	$\sim$	p	q
1	1	1	**1**	1	**0**	1	**1**
1	0	1	**1**	0	**0**	1	**0**
0	1	0	**1**	1	**1**	0	**1**
0	0	0	**0**	0	**1**	0	**0**

- The only rows that give us a false conclusion are the second and fourth ones. But in both cases, one of the premises is false.

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- An invalid form: denying the antecedent.
- We’ll show that $p \supset q, \sim p \not\models \sim q$

p	q	p	$\supset$	q	$\sim$	p	$\sim$	q
1	1	1	**1**	1	**0**	1	**0**	1
1	0	1	**0**	0	**0**	1	**1**	0
0	1	0	**1**	1	**1**	0	**0**	1
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