ELEMENTS OF DEDUCTIVE LOGIC

8. Gaps & Gluts: overview and motivations

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The story so far

- What logic is and why study it
- Basic concepts:
  - arguments and standard form
  - deductive validity of arguments
  - soundness of arguments
  - sentences, connectives and subsentences
  - sentential forms of arguments
- A syntax for a formal language ($\mathcal{L}_S$).

  $\Rightarrow$ representation of sentential form of natural language sentences with common *truth-functional connectives* (and, or, if…then, etc.).

The story so far (ctd.)

- A semantics giving possible assignments of truth values (here 0 or 1) to the sentences of $\mathcal{L}_S$.

  These assignments are constrained by the truth tables for the connectives.
- A simple, sound and complete tableau-based proof system to quickly establish validity of arguments in $\mathcal{L}_S$.

A glimpse ahead: reasoning with ‘gaps’ and ‘gluts’

- Philosophers sometimes claim that some sentences are
  - *neither* true *nor* false (truth value *gap*)
  - *both* true *and* false (truth value *glut*)
- Our so-called ‘classical’ (aka ‘bivalent’) semantics doesn’t accommodate these possibilities.
- I’ll present:
  - some motivation for the above claims,
  - a set of alternative semantics for $\mathcal{L}_S$,
  - associated sound and complete tableau-based proof systems.

A glimpse ahead: predicate logic

- The validity of *many* arguments isn't down to their sentential form!

Hippos

All hippos are bad-tempered.
Harry is a hippo.
Therefore Harry is bad-tempered.

(Most descriptive s. form: p, q, therefore r)

- They are valid because of their *sub-sentential* forms: we need to look 'inside' the atomic sentences.
- I'll present a relevant language ($\mathcal{L}_P$), with an associated semantics and tableau-based proof system.

A glimpse ahead: predicate logic (ctd.)

- We'll also look at reasoning about:
  - sentences asserting *identity* ('Clark Kent is Superman.'),
  - sentences involving *definite descriptions* ('The tallest man on earth lives in China.')

A glimpse ahead: natural deduction

- Time-permitting, we'll finish off the course with a completely different approach to proof: *natural deduction*.
- This very well known method aims to reproduce the intuitive pattern of reasoning employed by real people.
- To keep things simple, we'll restrict our brief overview to its application to classical sentential logic.

Definite descriptions

- Consider an utterance of:

  (1) 'The present king of France is bald.'

- Surely not true. Consider:

  (1') 'It is true that the present king of France is bald.'

- False, then? But consider:

  (1") 'It is false that the present king of France is bald.'

- Strawson (1950, p. 331): (1)–(1") are *neither true nor false*.
- As we shall see later, Russell (1905) disagrees, claiming (1) and (1') are false and (1") is true.

Universal quantification

- Consider:

  (2) 'All flying turtles fly South in the Winter.'

- True? False?
- Strawson (1952, p. 174) says: '*neither true nor false*, as there are no flying turtles!'
- Aristotelian logic says: '*false*, as there are no flying turtles!'
- Modern logic says: '*true*, as there are no flying turtles!'
- More on modern view later in the course.

Aspectual and factive verbs

- Consider:

  (3) 'Elvis Presley has stopped emailing me.'
  (4) 'John knows that 2 + 2 = 5.'

- True? False?
- Many philosophers & linguists say: 'neither'.
- (1), (2), (3) and (4) are often said to carry *presuppositions*.
- In case the sentences are true, so too are the presuppositions.
- In case the presuppositions are false, it is claimed that the sentences are neither true nor false.

Indicative conditionals

- Consider:

  (5) 'If I were dead, then my mother *would* be happy.'

  Everyone would (hopefully) agree: intuitively false.

- More contentious:

  (6) 'If I *am* dead, then my mother *is* happy.'

- Some feel strongly that (6) is neither true nor false (von Wright 1957, p. 130; De Finetti 1936, p. 35;...).
- In support, note the typical response to (6):

  (7) 'But you are not dead...'

  (Compare: 'But France doesn't have a king at present...')

Indicative conditionals (ctd.)

- Classical view: (6) is true since the logic of indicative conditionals is given by the truth table of '$\supset$'.
- 'Deviant' view: the logic of indicative conditionals is given by by the truth table of '$\rightarrow$'.

φ	ψ	φ	$\supset$	ψ	φ	$\rightarrow$	ψ
1	1		1			1	
1	0		0			0	
0	1		1			i	
0	0		1			i	

- Note: table for $\rightarrow$ is incomplete, since i-values for φ and ψ are not considered.

Indicative conditionals (ctd.)

- Paradoxes of material implication:
  - (i) $\psi \vDash \varphi \supset \psi$
  - (ii) $\neg\varphi \vDash \varphi \supset \psi$
  - (iii) $(\varphi \& \psi) \supset \chi \vDash (\varphi \supset \chi) \vee (\psi \supset \chi)$
  - (iv) $\vDash (\varphi \supset \psi) \vee (\psi \supset \chi)$
- Counterintuitive if we take '$\supset$' to stand for 'if…then'.
- Some good news here: the $\rightarrow$ counterparts of (i) and (ii) do *not* hold.

  (Countermodel: take φ false and ψ true.)
- For (iii) and (iv), it depends on the trivalent truth tables for $\vee$.

The sea battle tomorrow

- Consider:

  (8) 'There will be a sea battle tomorrow.'
- Claim: (8) is neither true nor false.
- Aristotle's argument, roughly put:
  1. If (8) were true or if it were false, this fact would be determined by the present state of the world.
  2. Nothing about the present state of the world could determine this. (Because of free will, quantum indeterminism,…)
  3. (8) is neither true nor false.
- Note: Aristotle concedes the truth of

  (9) 'Either there will be a sea battle tomorrow or there will not.'

Borderline cases

- Consider:

- Is it true that the central patch is orange? If not, is it *false* that it is orange?
- Some hold that we have a truth value gap: the patch is neither definitely orange, nor definitely not orange.
- This applies to many other 'vague' concepts aside from colour: being tall, being bald, being rich, being happy, etc.

'Liar' sentences

- Ok, so perhaps we can have truth value gaps…but truth value *gluts*!?!
- Consider:

  (10) 'This sentence is false.'
- A quick argument:
  1. Either (10) is true, or it is false.
  2. Assume that it is false. Then it is true, and hence both false and true.
  3. Assume that it is true. Then it is false, and hence both true and false.
  4. (10) is both true and false.
- One response is to deny 1: (10) is neither true nor false.
- We then have another argument for gaps.

Fancier 'liar' sentences

- Priest (2008, p. 130):

  (11) 'This sentence is either false or neither true nor false.'

- He then argues:
  1. Either (11) is true, or it is false, or it is neither true nor false.
  2. Assume that it is false. Then it is true. So it is both true and false. Therefore it is both true and not true.
  3. Assume that it is true. Then it is either false or neither true nor false. So it is both true and either false or neither true nor false. Therefore it is both true and not true.
  4. Assume that it is neither true nor false. Then it is true. So it is both true and neither true nor false. Therefore it is both true and not true.
  5. (11) is both true and not true.
- Note: we don't quite get the desired conclusion, i.e. that (11) is both true and false!

Sophie's choice

- Any potential cases of gluts more commonplace than Liar sentences?
- A moral quandary:

  Sophie must choose which one of her twins will be killed by the nazis and which one will be spared.
  If she fails to make up her mind, both will be murdered.
- According to some: it is both true and false that Sophie ought to save child #1.
- For formal argument, see McConnell (2010, section 4).

Next session

- Topic: some many-valued semantics
- *Optional* reading: Priest (2008), ch 7, if you can manage it (pretty tough for beginners).

References

- De Finetti, B. (1936). 'La logique de la probabilite', *Actes du congres international de philosophie scientifique, Fasc. IV*, Paris, Hermann, pp. 31-39.
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- Strawson, P.F. (1952). *Introduction to Logical Theory*, London: Methuen.
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