ELEMENTS OF DEDUCTIVE LOGIC

9. Gaps & Gluts: some semantics

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A general framework

- Last session: some reasons to countenance truth-value gaps and gluts.
- This time: how to suitably generalise sentential logic to make room for these cases.
- First, a more rigorous and general characterisation of a semantics...
- A set V of possible truth values.

Truth values

In classical logic: $V = \{0,1\}$.

A general framework (ctd.)

- A set $\{v_1, v_2, \ldots\}$ of possible valuations, assigning to each sentence a unique value in V.
- We *typically* start off with values for basic sentences, then derive values for more complex sentences.
- This is done by means of a set $\{f_\vee, f_\&, \ldots\}$ of truth functions for connectives of the language (represented by truth tables).
- These take truth values as inputs and provide a truth value as an output.

A general framework (ctd.)

Truth functions

- $f_\&(1,0) = 0$,
- $f_\vee(1,0) = 1$,
- $f_\sim(1) = 0$.

Values for complex sentences

Let $v(p) = 1$ and $v(q) = 0$. What is $v(\sim p \vee \sim q) = 1$?

$$\begin{aligned} v(\sim p \vee \sim q) &= f_\vee(v(\sim p), v(\sim q)) \\ &= f_\vee(f_\sim(v(p)), f_\sim(v(q))) \\ &= f_\vee(f_\sim(1), f_\sim(0)) \\ &= f_\vee(0,1) = 1 \end{aligned}$$

A general framework (ctd.)

- A set D of designated truth values.
- Used to define validity:

  An argument is valid iff there is no possible valuation that assigns to all premises a value in D but assigns to the conclusion a value that is *not* in D.

Designated values

In classical logic: $D = \{1\}$.
So, since $V = \{0, 1\}$:

An argument is classically valid iff there is no possible valuation that assigns to all premises a value of 1 but assigns to the conclusion a value of 0.

Introduction

- In L8: some possible cases of truth value gaps.
  - Conditionals
  - Presupposition failure
  - Future contingents
  - Borderline cases
- Let's look at some semantics for some logics to handle the last 3 kinds of cases (I leave conditionals to one side).
  - B3 and K3 (Kleene 1952),
  - SV (van Fraassen 1966).

The 'weak Kleene', aka 'Bochvar', logic B3

- We have $V = \{0, i, 1\}$ and $D = \{1\}$.
- And an expanded set of truth tables:

$f_{\sim}$	
1	0
0	1
i	i

$f_{\vee}$	1	0	i
1	1	1	i
0	1	0	i
i	i	i	i

$f_{\&}$	1	0	i
1	1	0	i
0	0	0	i
i	i	i	i

$f_{\supset}$	1	0	i
1	1	0	i
0	1	1	i
i	i	i	i

$\Rightarrow$ If one of the inputs is i, then so too is the output.

Comments on B3

- B3 is well-suited to presupposition failure: 'nonsense breeds nonsense' or 'garbage in, garbage out'.
  (1) 'All flying turtles fly south in the Winter and there are no flying turtles.'
  (2) ' Either the present king of France is bald or the queen of England is not.'
- Truth tables for B3: (1) and (2) are neither true nor false.
- Arguably seems intuitive.
- Not so good for future contingents and borderline cases:
  (3) 'I am a Somali pirate and there will be a sea battle tomorrow.'
  (4) ' Either James is bald or he is a banker'
- (3) seems false (right??)
- (4) seems true, assuming that James is borderline bald and that he is a banker.

The ‘strong Kleene’ logic K3

- Again: $V = \{0, i, 1\}$ and $D = \{1\}$.
- And another expanded set of truth tables:

$f_{\sim}$	
1	0
0	1
i	i

$f_{\vee}$	1	0	i
1	1	1	1
0	1	0	i
i	1	i	i

$f_{\&}$	1	0	i
1	1	0	i
0	0	0	0
i	i	0	i

$f_{\supset}$	1	0	i
1	1	0	i
0	1	1	1
i	1	i	i

$\Rightarrow$ The output is i, unless there are classical inputs that would classically suffice to determine a different output.

Comments on K3

- Is K3 any better than B3 for borderline cases? Future contingents?
- Some improvements:
  - (3) ‘I am a Somali pirate and there will be a sea battle tomorrow.’
  - (4) ‘ Either James is bald or he is a banker’
- (3) and (4) now come out false and true, respectively.
- But problems remain:
  - (5) ‘Either there will be a sea battle tomorrow or there won’t’
  - (6) ‘If the patch is orange, then it is orange’
- These seem to be tautologies. . .

Comments on K3 (ctd.)

- However (Priest 2008, p. 124):
  - $\nvDash_{K3} \varphi \vee \sim \varphi$ and (equivalently)
  - $\nvDash_{K3} \varphi \supset \varphi$.

  Countermodel: $v(\varphi) = i$
- Special case of a weird general feature of K3:

  For all sentences φ in $\mathcal{L}_S$, $\nvDash_{K3} \varphi$

  In other words: no tautologies in K3!
- Indeed:
  - For any sentence φ, if all its atomic subsentences have value i, then so too does φ.
  - So for any sentence φ, there is at least one valuation v such that $v(\varphi)$ is the non-designated value i.
- Note: this is also true of B3.

Comments on K3 (ctd.)

- Priest discusses the logic Ł3 (Łukasziewicz 1930), which just modifies the table for $\supset$, so that the equivalence between $\sim \varphi \vee \psi$ and $\varphi \supset \psi$ fails.
- In Ł3, there *are* tautologies, but $\varphi \vee \sim \varphi$ isn’t one of them.

Next session

- Topic: more on semantics for gaps and gluts.

References

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- van Fraassen, B. (1966). 'Singular terms, truth-value gaps and free logic'. *Journal of Philosophy*, 63:481-95.