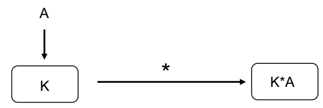
Conceptual and Practical Challenges in Belief Revision

Pavlos Peppas

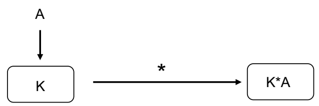
Overview



- Conceptual Challenges
  - Iterated Revision
  - Relevance-Sensitive Revision

- Practical Challenges
  - Computational Complexity
  - Representational Cost

AGM Belief Revision



(K*1) K*A is a theory

(K*2) $A \in (K*A)$

(K*3) $K*A \subseteq K+A$

(K*4) If $\neg A \notin K$ then $K+A \subseteq K*A$

(K*5) If A is consistent then K*A is consistent

(K*6) If $A \equiv B$ then $K*A = K*B$

(K*7) $K*(A \wedge B) \subseteq (K*A)+B$

(K*8) If $\neg B \notin (K*A)$ then $(K*A)+B \subseteq K*(A \wedge B)$

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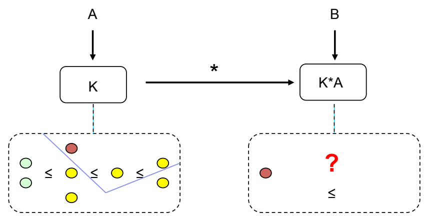

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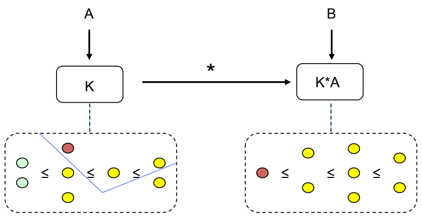
Iterated Belief Revision



- Spohn's OCF
- Darwiche and Pearl Postulates

(K*1) K*A is a theory

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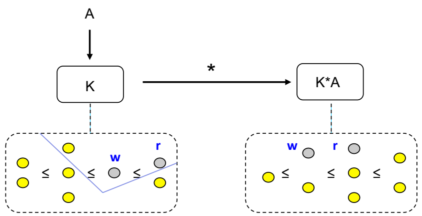
Iterated Revision – The Darwiche and Pearl Approach



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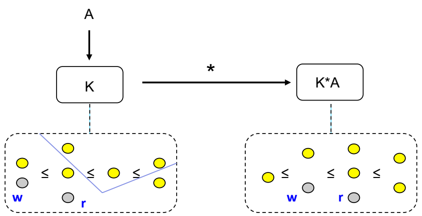

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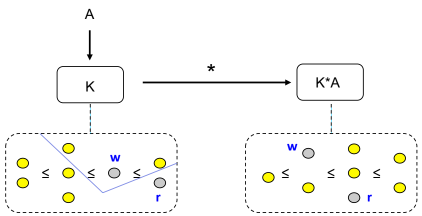

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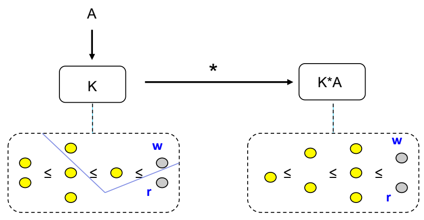

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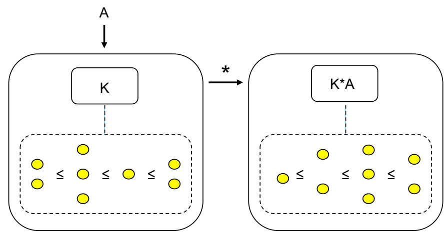
The DP Approach: What's a Belief State?



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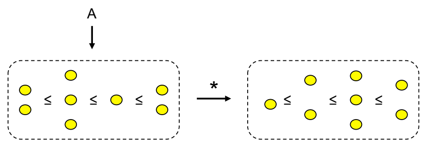

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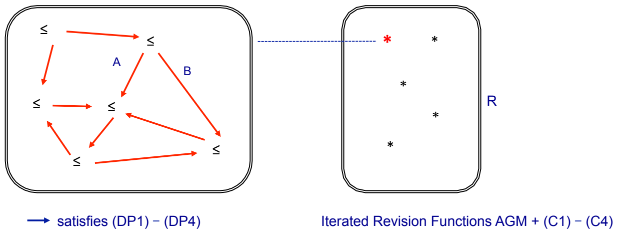

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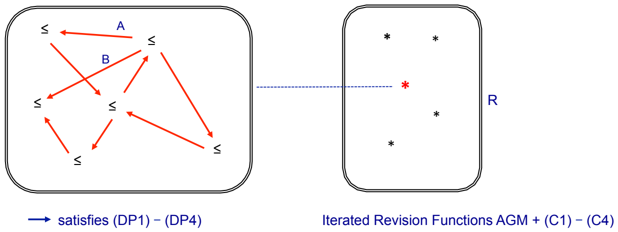

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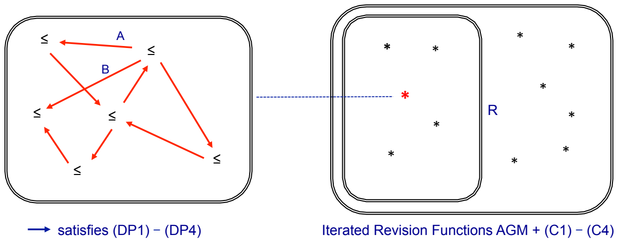
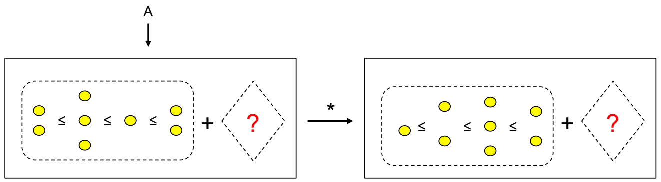


THEOREM:

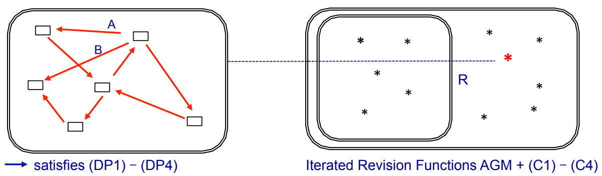
There exists revision functions, that satisfies the DP constraints, which do not belong to R.

The DP Approach: What's a Belief State?



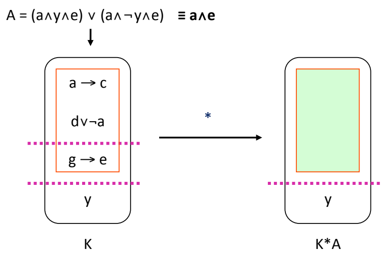
OBSERVATION:

DP belief states are MORE than just preorders over worlds. So what's a "DP-complete" state?



Relevance-Sensitive Revision

Parikh's Approach for Relevance-Sensitive Revision

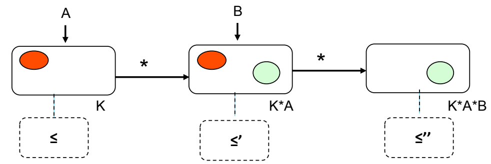


(wP) If $K = Cn(X,Y)$, $L_X \cap L_Y = \emptyset$ and $A \in L_X$, then $(K*A) \cap L_Y = K \cap L_Y$.

⇕

(SP) If $Diff(K,r) \subset Diff(K,z)$ then $r < z$.

The DP approach is Inconsistent with (P)



THEOREM:
Let K be *any* splittable theory. There exists a K-faithful preorder over worlds ≤ and a sentence A such that *every* posterior preorder ≤' that is faithful to K*A and satisfies (the semantic constraints for) axiom (wP), violates each one of the constraints (DP1) – (DP4).

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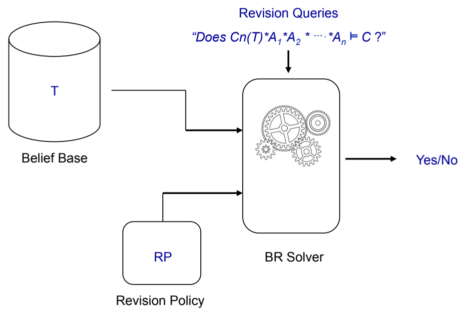
Let K be *any* splittable theory. For *any* K-faithful preorder over worlds ≤ there exists a sentence A such that *every* posterior preorder ≤' that is faithful to K*A and satisfies (the semantic constraints for) axiom (wP), violates (DP2).

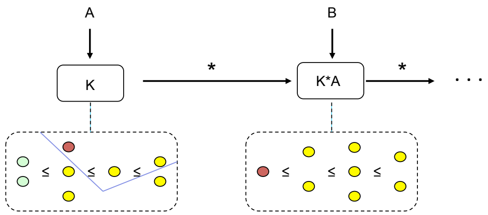
THEOREM:
Dalal's operator satisfies (wP) but violates the DP postulates.

Practical Challenges

Building a Scalable Belief Revision Solver



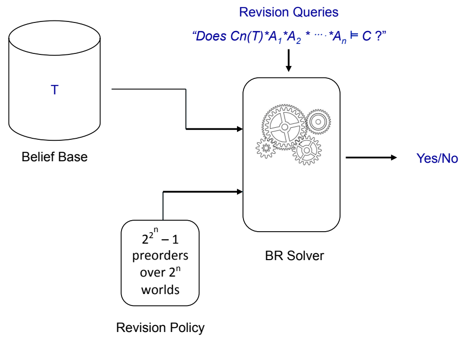
Representational Cost of a Revision Function



For a propositional language built over n propositional variables there exist: $\mathbf{2^n}$ possible worlds, and hence $\mathbf{2^{2^n} - 1}$ consistent theories. Thus to specify a (iterated) revision function we need to explicitly provide $2^{2^n} - 1$ preorders over the 2^n possible worlds.

*i.e. for a language with just **9 variables**, the number of preorders we need to specify is 2^{2048}-1, which is **more than the atoms in the observable universe!***

Building a Scalable Belief Revision Solver



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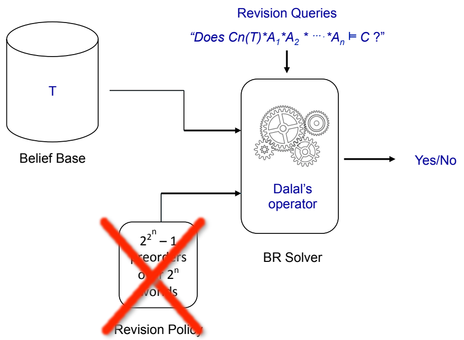
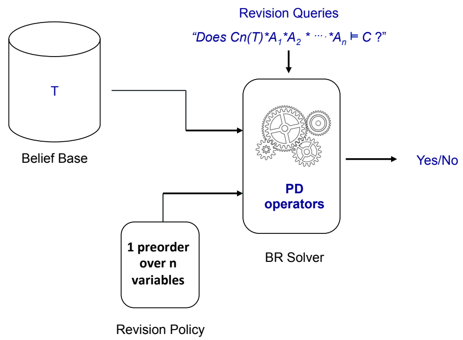


Too restrictive!

Building a Scalable Belief Revision Solver



Dalal Preorders

P: the set of all propositional variables (or atoms) of the language.

Diff(w,r) = $\{ x \in P: w \models x \text{ and } r \not\models x\} \cup \{ x \in P: w \not\models x \text{ and } r \models x\}$.

Dalal Preorder: $r \leq r'$ iff for some $w \in [K]$, $|Diff(w,r)| \leq |Diff(w',r')|$, for all $w' \in [K]$.

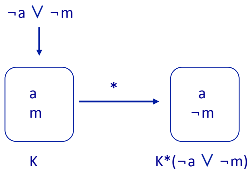
$$abc \;\leq\; \{\neg abc,\; a\neg bc,\; ab\neg c\} \;\leq\; \{a\neg b\neg c,\; \neg ab\neg c,\; \neg a\neg bc\} \;\leq\; \neg a\neg b\neg c$$

Counterexample

A circuit consists of an adder and a multiplier.

- a : “the adder is working”
- m: “the multiplier is working”

Adders are more reliable than multipliers and hence :



Dalal Preorder: $a\,m \;\leq\; \{\neg a\, m,\; a\,\neg m\} \;\leq\; \neg a\,\neg m$ $\Rightarrow$ $K*(\neg a \vee \neg m) \equiv (\neg a \wedge m) \vee (a \wedge \neg m)$

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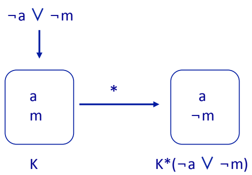

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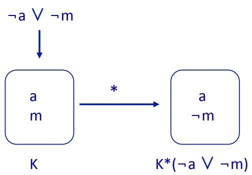

m ⊲ a

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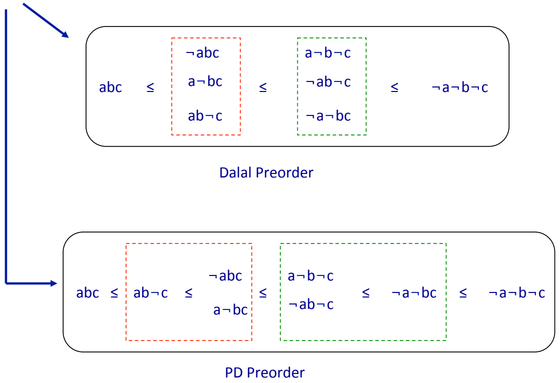
PD Preorder: $a\,m \;\leq\; a\,\neg m \;\leq\; \neg a\, m \;\leq\; \neg a\,\neg m$ $\Rightarrow$ $K*(\neg a \vee \neg m) \equiv (a \wedge \neg m)$ ✓

Example

c ◁ a ≈ b

K ≡ a ∧b ∧c



Parameterised Difference Preorders

Let ⊴ be a total preorder over atoms. For a set of atoms S, and an atom y, define S_y = { x ∈ S: x ⊴ y }.

The PD preorder ≤ generated from ⊴ that is assigned to a theory K is defined as follows:

r ≤ r′ iff there is a w ∈K such that for all w′∈K one of the following holds:

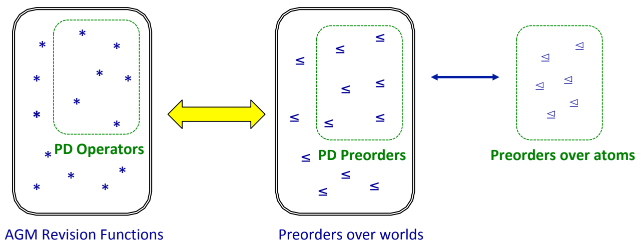
a. |Diff(w,r)| < |Diff(w′,r′)|

b. $|Diff(w,r)_y|$ = $|Diff(w',r')_y|$, for all atoms y.

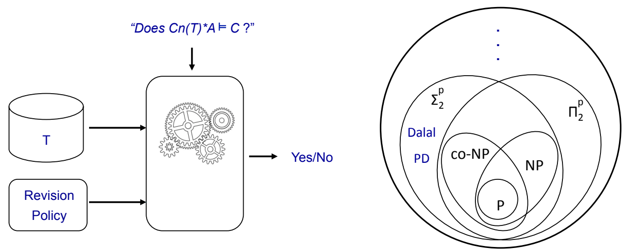
c. |Diff(w,r)| = |Diff(w′,r′)|, and for some atom y, $|Diff(w,r)_y|$ > $|Diff(w',r')_y|$ and moreover

$|Diff(w,r)_q|$ = $|Diff(w',r')_q|$, for all q ◁ y

Axiomatic Characterization of PD Operators

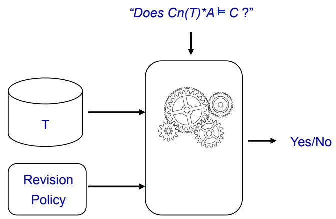


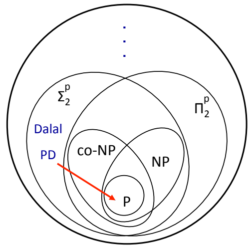
Computational Complexity of a BR Solver



- Dalal's operator belongs to the second layer of the polynomial hierarchy (needs at most *log(n)* calls to an NP-oracle).
- PD Operators also belong to the second layer of the polynomial hierarchy (needs at most *n*^{½} calls to an NP-oracle).

Computational Complexity of a BR Solver





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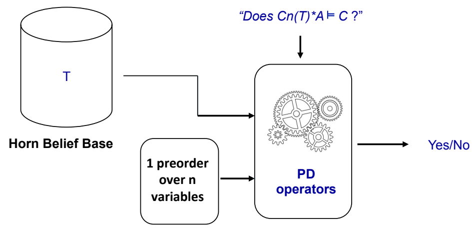
- PD Operators also belong to the second layer of the polynomial hierarchy (needs at most $n^{½}$ calls to an NP-oracle).
- If T is Horn, and the query is Horn and of bounded length, then PD operators can be processed in **linear time**.

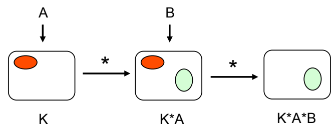
A Scalable Belief Revision Solver



- Increase expressivity (e.g. go beyond Horn belief bases, or allow the atoms-preorder to vary).
- Use **domain-specific information on relevance**.to (partially) build the atoms-preorder.

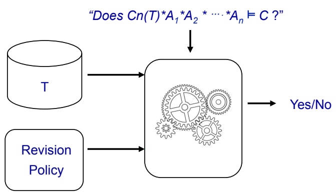
Conclusion

Conceptual Challenges:



- A DP-complete definition of belief states.
- A domain-specific account of relevance-sensitive revision.
- Reconcile iteration and relevance.

Practical Challenges:



Trade generality for efficiency in order to deal with:

- Representational cost.
- Computational complexity.