Abstract

Forgetting is a central task of belief management which has many facets. However, there is no commonly accepted answer the quest on how different forms of forgetting could be captured as (belief) change. Starting from a high-level commonsense perspective on forgetting, this talk presents different forms of forgetting. In the course of the talk, we focus on the aspects of gradualness. Decrement operators are presented, which extend iterated contraction by the aspect of gradual change.

Forgetting and Gradual Iterated Contraction

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Introduction: Forgetting

(Logical) Forgetting

Forgetting a Fact (Lin & Reiter, 1994, AAAI)
For a (first-order) formula φ and an ground atom p

$$\mathit{forget}(\varphi, p) = \varphi_p^+ \vee \varphi_p^-$$

where $\varphi_p^+ = \varphi[p/\top]$ and $\varphi_p^- = \varphi[p/\bot]$.

Variable Forgetting (Lang, Liberatore, Marquis, 2003, JoAIR) For a (propositional) formula φ, an variable σ

$$\mathit{forget}_{var}(\varphi, \sigma) = \varphi_\sigma^+ \vee \varphi_\sigma^-$$

- Boole[1]: *"eliminations of middle terms"*
- very successful (extended to ASP, DL, ...)
- technical notion of forgetting

[1] pointed out by Zhang in 2009

(Common Sense) Forgetting

A more commonsensical view on forgetting [BKS+19]:

- Contraction
  Lose/Delete a information/belief α
- Ignorance
  Giving up the judgement on α
- Conditionalization
  Restriction to a specific context
- Marginalisation
  Remove the aspect α from the language

- Focussing
- Tunnel View
- Abstraction
- Fading Out

Thursday, 29th August 2019:
KRR-Track

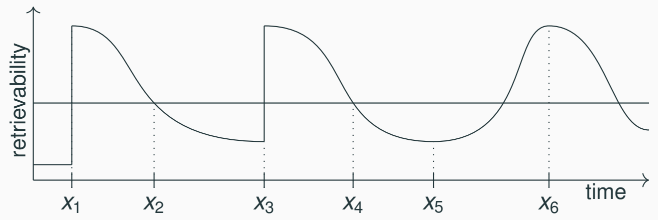
(Common Sense) Forgetting

	Tunnel View	Focussing	Abstraction
Periphrasis	Temporary Restriction of abilities	Concentration on a case	Condense information of individuals
Example	Stress situation	Doctor's examination	extracting concepts
(Possible) Technical Manifestations	• Restriction to Sub-Signature Σ' (selected by Accessibility) • Restricted or Limited Inference $\approx^r$	Restriction to Sub-Signature Σ' (selected by relevance)	deductive/inductive reasoning

Fading Out

Fading Out is the kind of change, where beliefs gradually fade away over the time.

Example: The meal on this morning



We investigate a realisation within the framework of (iterated) belief change.

Background on Belief Change

The setting for (iterative) belief change: Prelim & Notions

$\mathcal{L} = \{\alpha, \beta, \ldots\}$ a propositional language
$\Omega = \{\omega, \omega_1, \omega_2, \ldots\}$ set of worlds (interpretations)
$[\![\alpha]\!]$ models of α
$\min(X, \leq)$ denotes the set of minimal elements from X w.r.t. $\leq$.

Epistemic States
Each agent is equipped with an *epistemic state*.

$\mathcal{E} = \{\Psi, \Psi_1, \Psi_2, \ldots\}$ set of epistemic states
$Bel(\Psi)$ a belief set[2] for each epistemic state
$[\![\Psi]\!] = [\![Bel(\Psi)]\!]$ models of the beliefs

A mapping $\Psi \mapsto \leq_\Psi$ is a *faithful assignment* [DP97] if $\min(\Omega, \leq_\Psi) = [\![\Psi]\!]$.

Belief Change Operations
A *belief change operation* is a function $\circ : \mathcal{E} \times \mathcal{L} \to \mathcal{E}$.

A faithful assignment $\Psi \mapsto \leq_\Psi$ is called strong [KP08] w.r.t. $\circ$ if

$$\text{if } \alpha_1 \equiv \beta_1, \ldots, \alpha_n \equiv \beta_n, \text{ then } \leq_{\Psi \circ \alpha_1 \circ \ldots \circ \alpha_n} = \leq_{\Psi \circ \beta_1 \circ \ldots \circ \beta_n}$$

[2]deductively closed set of formula

Belief Change: Contraction

Postulates for AGM contraction on epistemic states [CGMW08, KP17]:

$$Bel(\Psi - \alpha) \subseteq Bel(\Psi) \quad \text{(C1)}$$

$$\text{if } \alpha \notin Bel(\Psi), \text{ then } Bel(\Psi) \subseteq Bel(\Psi - \alpha) \quad \text{(C2)}$$

$$\text{if } \alpha \not\equiv \top, \text{ then } \alpha \notin Bel(\Psi - \alpha) \quad \text{(C3)}$$

$$Bel(\Psi) \subseteq Cn(Bel(\Psi - \alpha) \cup \{\alpha\}) \quad \text{(C4)}$$

$$\text{if } \alpha \equiv \beta, \text{ then } Bel(\Psi - \alpha) = Bel(\Psi - \beta) \quad \text{(C5)}$$

$$Bel(\Psi - \alpha) \cap Bel(\Psi - \beta) \subseteq Bel(\Psi - (\alpha \wedge \beta)) \quad \text{(C6)}$$

$$\text{if } \beta \notin Bel(\Psi - (\alpha \wedge \beta)), \text{ then } Bel(\Psi - (\alpha \wedge \beta)) \subseteq Bel(\Psi - \beta) \quad \text{(C7)}$$

Theorem ([CGMW08, KP17])
A belief change operator $-$ satisfies (C1) *to* (C7) *iff there is a faithful assignment $\Psi \mapsto \leq_\Psi$ such that:*

$$[\![\Psi - \alpha]\!] = [\![\Psi]\!] \cup \min([\![\neg\alpha]\!], \leq_\Psi)$$

Iterative Belief Change: Contraction

The following semantic principles for where proposed for iterated contraction [CGMW08, KP17]:

$$\text{if } \omega_1, \omega_2 \in [\![\alpha]\!], \text{ then } \omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi - \alpha} \omega_2$$

$$\text{if } \omega_1, \omega_2 \in [\![\neg\alpha]\!], \text{ then } \omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi - \alpha} \omega_2$$

$$\text{if } \omega_1 \in [\![\neg\alpha]\!] \text{ and } \omega_2 \in [\![\alpha]\!], \text{ then } \omega_1 <_\Psi \omega_2 \Rightarrow \omega_1 <_{\Psi - \alpha} \omega_2$$

$$\text{if } \omega_1 \in [\![\neg\alpha]\!] \text{ and } \omega_2 \in [\![\alpha]\!], \text{ then } \omega_1 \leq_\Psi \omega_2 \Rightarrow \omega_1 \leq_{\Psi - \alpha} \omega_2$$

Weak Decrement Operators

Towards more gradually belief contraction: hesitant contraction

An important generalisation of AGM contraction is the weakening of success:

$$\alpha \notin \mathit{Bel}(\Psi \circ \alpha) \text{ or } \mathit{Bel}(\Psi) = \mathit{Bel}(\Psi \circ \alpha) \qquad \text{(relative sucess)}$$

We generalise contraction in another way...

Definition (hesitant contraction)
A belief change operator $\circ$ is called a *hesitant contraction operator* if the following postulate is fulfilled[3]:

$$\text{if } \alpha \not\equiv \top, \text{ then there exists } n \in \mathbb{N}_0 \text{ such that } \alpha \notin \mathit{Bel}\left(\Psi \circ^n \alpha\right) \qquad \text{(hesitant success)}$$

If $\circ$ is an hesitant contraction operator, then we define a corresponding operator $\bullet$ by

$$\Psi \bullet \alpha = \Psi \circ^n \alpha,$$

where $n = 0$ if $\alpha \equiv \top$, otherwise n is the smallest integer such that $\alpha \notin \mathit{Bel}(\Psi \circ^n \alpha)$.

[3] Define $\Psi \circ^0 \alpha = \Psi$ and $\Psi \circ^n \alpha = (\Psi \circ^{n-1} \alpha) \circ \alpha$.

Adopt AGMes contraction

We adopt the postulates for AGM contraction for decrementing beliefs.

Definition (Weak Decrement Operator)
A belief change operator $\circ$ is called a *weak decrement operator* if the following postulates are fulfilled:

$$\mathit{Bel}(\Psi \bullet \alpha) \subseteq \mathit{Bel}(\Psi) \qquad \text{(D1)}$$

$$\text{if } \alpha \notin \mathit{Bel}(\Psi), \text{ then } \mathit{Bel}(\Psi) \subseteq \mathit{Bel}(\Psi \bullet \alpha) \qquad \text{(D2)}$$

$$\circ \text{ is a hesitant contraction operator} \qquad \text{(D3)}$$

$$\mathit{Bel}(\Psi) \subseteq \mathit{Cn}(\mathit{Bel}(\Psi \bullet \alpha) \cup \{\alpha\}) \qquad \text{(D4)}$$

$$\text{if } \alpha_1 \equiv \beta_1, \ldots, \alpha_n \equiv \beta_n, \text{ then } \mathit{Bel}(\Psi \circ \alpha_1 \circ \ldots \circ \alpha_n) = \mathit{Bel}(\Psi \circ \beta_1 \circ \ldots \circ \beta_n) \qquad \text{(D5)}$$

$$\mathit{Bel}(\Psi \bullet \alpha) \cap \mathit{Bel}(\Psi \bullet \beta) \subseteq \mathit{Bel}(\Psi \bullet (\alpha \wedge \beta)) \qquad \text{(D6)}$$

$$\text{if } \beta \notin \mathit{Bel}(\Psi \bullet (\alpha \wedge \beta)), \text{ then } \mathit{Bel}(\Psi \bullet (\alpha \wedge \beta)) \subseteq \mathit{Bel}(\Psi \bullet \beta) \qquad \text{(D7)}$$

The postulates (D1) to (D7) correspond to (C1) to (C7).

Weak Decrement Operators

Theorem (Representation Theorem)

Let $\circ$ be a belief change operator.

Then the following items are equivalent:

(a) *$\circ$ is a weak decrement operator (i.e. fulfils* (D1) *to* (D7)*)*
(b) *there exists a strong faithful assignment $\Psi \mapsto \leq_\Psi$ with respect to $\circ$ such that:*

$$\text{there exists } n \in \mathbb{N}_0 \text{ such that } [\![\Psi \circ^n \alpha]\!] = [\![\Psi]\!] \cup \min([\![\neg\alpha]\!], \leq_\Psi)$$
$$\text{and } n \text{ is the smallest integer such that } [\![\Psi \circ^n \alpha]\!] \not\subseteq [\![\alpha]\!] \qquad \text{(decrement success)}$$

Weak Decrement Operators: Properties

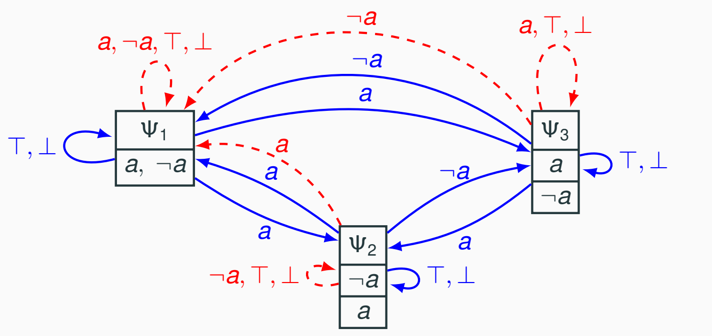
Corollary
If $\circ$ is a weak decrement operator, then $\bullet$ fulfils (C1) *to* (C7).

However, there are examples such that $\circ$ violates *inclusion* (C1), i.e.

$$[\![\Psi]\!] \not\subseteq [\![\Psi \circ \alpha]\!] \qquad \Psi_1 \circ a = \Psi_2$$

and examples such that

$$[\![\Psi \circ \alpha]\!] \not\subseteq [\![\Psi]\!] \cup \min([\![\neg\alpha]\!], \leq_\Psi) \qquad \Psi_3 \circ a = \Psi_2$$



Cautious Decrement Operators

Definition

A $\circ$ weak decrement operator is called a *cautious decrement operator*, if the following postulates are satisfied:

$$Bel(\Psi \circ \alpha) \subseteq Bel(\Psi) \qquad \text{(D1s)}$$

$$Bel(\Psi \bullet \alpha) \cap Bel(\Psi \bullet \beta) \subseteq Bel(\Psi \circ (\alpha \wedge \beta)) \qquad \text{(D6s)}$$

Representation theorem:

Theorem (Representation Theorem)

Let $\circ$ be a belief change operator. Then the following is equivalent:

(a) *$\circ$ is an cautious decrement operator*

(b) *there exists a strong faithful assignment $\leq_\Psi$ with respect to $\circ$ such that* (decrement success) *and the following is satisfied:*

$$[\![\Psi]\!] \subseteq [\![\Psi \circ \alpha]\!] \qquad \text{(SFApart1)}$$

$$[\![\Psi \circ \alpha]\!] \subseteq [\![\Psi]\!] \cup \min([\![\neg\alpha]\!], \leq_\Psi) \qquad \text{(SFApart2)}$$

Decrement Operators

Decrement Operators: In the Semantic of Total Preorders

A restricted version of the strong faithful assignments.

Definition (Decreasing Assignment)

Let $\circ$ be a hesitant belief change operator. A strong faithful assignment $\Psi \mapsto \leq_\Psi$ with respect to $\circ$ is said to be a *decreasing assignment* (with respect to $\circ$) if the following postulates are satisfied:

if $\omega_1, \omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR8)

if $\omega_1, \omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR9)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Rightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR10)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 <_\Psi \omega_2 \Rightarrow \omega_1 <_{\Psi\circ\alpha} \omega_2$ (DR11)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_2 \ll_\Psi \omega_1 \Rightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR12)

if $\omega_1 \in [\![\neg\alpha]\!]$, $\omega_2 \in [\![\alpha]\!]$ and $\omega_2 \leq_\Psi \omega_3$ for all ω_3, then $\omega_2 \leq_{\Psi\circ\alpha} \omega_1$ (DR13)

$\omega_2 \ll_\Psi \omega_1$ iff $\omega_2 <_\Psi \omega_1$ and there is no ω_3 s.t. $\omega_2 <_\Psi \omega_3 <_\Psi \omega_1$

Example in the TPO model

Initial

Ψ: α	$\overline{\alpha}$
	ω_1
ω_2	
ω_x	ω_y

Possible Outcomes ($\Psi \circ \alpha$)

Ψ': α	$\overline{\alpha}$
ω_2	ω_1
ω_x	ω_y

Ψ'': α	$\overline{\alpha}$
ω_2	
	ω_1
ω_x	ω_y

if $\omega_1, \omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR8)

if $\omega_1, \omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR9)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Rightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR10)

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if $\omega_1 \in [\![\neg\alpha]\!]$, $\omega_2 \in [\![\alpha]\!]$ and $\omega_2 \leq_\Psi \omega_3$ for all ω_3, then $\omega_2 \leq_{\Psi\circ\alpha} \omega_1$ (DR13)

Decrement Operators: Syntactic counterparts for (DR8) and (DR9)

In the light of (D1) – (D7)

if $\omega_1, \omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR8)

if $\omega_1, \omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR9)

correspond to (D8) and (D9):

if $\neg\alpha \models \beta$, then $Bel(\Psi \circ \alpha \bullet \beta) =_\alpha Bel(\Psi \bullet \beta)$ (D8)

if $\alpha \models \beta$, then $Bel(\Psi \circ \alpha \bullet \beta) =_{\neg\beta} Bel(\Psi \bullet \beta)$ (D9)

where $=_\alpha$ is called α-equivalence, defined by:

$$X =_\alpha Y \text{ if } [\![X]\!] \cap [\![\alpha]\!] = [\![Y]\!] \cap [\![\alpha]\!]$$

Intuitively, $X =_\alpha Y$ if both set "agree" on everything about α.

Example
$X = Cn(bird \wedge flys, penguin \rightarrow flys)$ and $Y = Cn(bird \wedge flys, penguin \rightarrow \neg flys)$.
$X =_{bird \wedge \neg penguin} Y$, but $X \neq_{bird} Y$.

Decrement Operators: Syntactic counterparts (DR13)

Define the following order:

$$\alpha \preceq^\circ_\Psi \beta \text{ if } Bel(\Psi \bullet \alpha \wedge \beta) \subseteq Bel(\Psi \bullet \alpha)$$

Intuitively, $\alpha \preceq^\circ_\Psi \beta$ means that in the state Ψ the agent is more (or equally) willing to remove the belief α than the belief β.

With $\prec^\circ_\Psi$ we denote the strict variant of $\preceq^\circ_\Psi$ and define $\alpha \lll^\circ_\Psi \beta$ if $\alpha \prec^\circ_\Psi \beta$ and there is no γ such that $\alpha \prec^\circ_\Psi \gamma \prec^\circ_\Psi \beta$.

In the light of (D1) – (D7)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_2 \ll_\Psi \omega_1 \Rightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR12)

the postulate (DR12) corresponds to:

if $\alpha \models \beta$ and $\neg\alpha \models \gamma$, then $\gamma \lll^\circ_\Psi \beta \Rightarrow \beta \preceq^\circ_{\Psi\circ\alpha} \gamma$ (D12)

Decrement Operators: Syntactic counterparts

In the light of (D1) – (D7)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Rightarrow \omega_1 \leq_{\Psi\circ\alpha} \omega_2$ (DR10)

if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 <_\Psi \omega_2 \Rightarrow \omega_1 <_{\Psi\circ\alpha} \omega_2$ (DR11)

corresponds to

if $\alpha \models \gamma$, then $\Psi \circ \alpha \bullet \beta \models \gamma \Rightarrow \Psi \bullet \beta \models \gamma$ (D10)

if $\neg\alpha \models \gamma$, then $\Psi \bullet \beta \models \gamma \Rightarrow \Psi \circ \alpha \bullet \beta \models \gamma$ (D11)

Additionally, the support postulate (D13) axiomatically enforces that a single step does not add any beliefs. This corresponds to:

$$Bel(\Psi \circ \alpha) \subseteq Bel(\Psi) \quad \text{(D13)}$$

Decrement Operators and a Representation Theorem

Definition (Decrement Operator)

A $\circ$ weak decrement operator is said to be a *decrement operator* if $\circ$ satisfies (D8) to (D13).

The following theorem connections operators and the semantically model.

Theorem (Representation Theorem)

Let $\circ$ be a belief change operator. Then the following items are equivalent:

(a) *$\circ$ is a decrement operator*

(b) *there exists a decreasing assignment $\Psi \mapsto \leq_\Psi$ with respect to $\circ$ that satisfies* (decrement success)*, i.e.:*

there exists $n \in \mathbb{N}_0$ such that $[\![\Psi \circ^n \alpha]\!] = [\![\Psi]\!] \cup \min([\![\neg\alpha]\!], \leq_\Psi)$
and n is the smallest integer such that $[\![\Psi \circ^n \alpha]\!] \not\subseteq [\![\alpha]\!]$

Properties of Decrement Operators

Proposition

If $\circ$ is a decrement operator and $\Psi \mapsto \leq_\Psi$ a corresponding decreasing assignment, then the following postulate are satisfied:

$$[\![\Psi]\!] \subseteq [\![\Psi \circ \alpha]\!] \subseteq [\![\Psi]\!] \cup \min([\![\neg\alpha]\!], \leq_\Psi) \qquad \text{(partial success)}$$

$$\alpha \notin Bel(\Psi \circ \alpha) \text{ or } Bel(\Psi) = Bel(\Psi \circ \alpha) \qquad \text{(relative sucess)}$$

Furthermore:

- Decrement Operators are not unique (in contrast to Improvement Operators [KP08])
- Decrement Operators are a strict subclass of Weak Decrement Operators.

Fin

(Open) Questions (?)

Decrement Operators

- Isn't there something similar for revision?
  Yes... Improvement Operators (Konieczny and Pino Pérez, 2009)
  Relation not examined... yet
- Examine subclasses of (weak) decrement operators
- What is the nature of $\preceq_\Psi^\circ$?
  It is no epistemic entrenchment

Forgetting:

- There are more "types" of forgetting?
- Common Framework for forgetting?
- Finding natural realisations for all "types" of forgetting

A Tool for Model Checking

I am also interested in model checking belief changes.

Just for advertising:
`https://www.fernuni-hagen.de/wbs/alchourron/`

Fin

Thank you for your attention!

- Forgetting
- Belief Change
- Hesitant and Weak Decrement Operators
- Decrement Operators
- Properties

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